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TECHNICAL NOTE R-48

A FORTRAN PROGRAM FOR THREE-DEGREE OF FREEDOM TRAJECTORIES, REFERENCED TO GEOCENTRIC COORDINATES AND TO AN ARBITRARY POINT ON THE EARTH'S SURFACE

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Prepared By

H. M. Minshew

May, 1963

BROWN ENGINEERING COMPANY INC.
HUNTSVILLE, ALABAMA

A FORTRAN PROGRAM FOR THREE-DEGREE OF FREEDOM TRAJECTORIES, REFERENCED TO GEOCENTRIC COORDINATES AND TO AN ARBITRARY POINT ON THE EARTH'S SURFACE

May, 1963

Prepared Under the Direction Of

PHYSICAL SCIENCES LABORATORY
ARMY MISSILE COMMAND
REDSTONE ARSENAL, ALABAMA
(AMM Systems Division Scope of Work SW-Z-48-61)

By

SCIENTIFIC RESEARCH LABORATORIES
BROWN ENGINEERING COMPANY, INC.

Technical Note R-48

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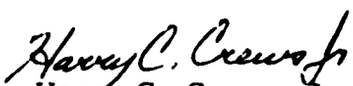
ABSTRACT

This report describes a FORTRAN II computer program for generating synthetic ballistic vehicle trajectories. The vehicle is considered to have a constant ballistic coefficient and to be under the influence of gravitational, aerodynamic, centrifugal and Coriolis forces. The program contains provisions for computing the trajectory in the reference frame of an arbitrarily located radar station.

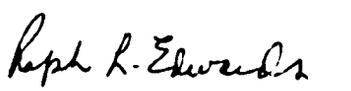
A copy of the program may be obtained from the Scientific Programming Library, Program No. SP-66.

Approved By:



Harry C. Crews, Jr.
Director, Electromagnetics Laboratory
Brown Engineering Company, Inc.

Approved By:



Ralph L. Edwards
Chief, Hypervelocity Physics Branch
AMC Physical Sciences Laboratory

LIST OF SYMBOLS

A	Vehicle reference area
A_z	Vehicle azimuth angle (measured in radar system)
C_D	Drag coefficient (a characteristic of the vehicle)
E	Vehicle elevation angle (measured in radar system)
g_o	Acceleration of gravity at sea-level
H	Height above the earth's surface
k	A force constant defined on page 4
$\vec{i}, \vec{j}, \vec{k}$	Unit vectors along the X, Y, Z axes respectively of the earth-fixed coordinate system
$\vec{R}$	Radius vector from earth's center to the vehicle
$\vec{R}_o$	A previous value of $\vec{R}$ used to compute ground range
R_l	Slant range to vehicle measured from radar site
$\vec{R}_l$	R_l in vector form
$\dot{R}_l$	Time rate of change of the slant range
R_e	Mean radius of the earth
$R_e(\phi)$	Radius of the earth at latitude ϕ
R_{e_r}	Radius of the earth at the latitude of the radar site
S	Ground range defined on page 7
ΔS	An increment of ground range
t	Time
Δt	An increment of time

LIST OF SYMBOLS (cont.)

$\vec{V}$	Vector velocity of the vehicle in the earth-fixed X, Y, Z system
V	Magnitude of $\vec{V}$
V_{x_m}, V_{y_m}, V_{z_m}	Velocity components in the local reference system of the vehicle
W	Weight of the vehicle
X, Y, Z	Earth-fixed coordinate system (Figure 1)
$\dot{X}$, $\dot{Y}$, $\dot{Z}$	First time derivatives of X, Y, Z
$\ddot{X}$, $\ddot{Y}$, $\ddot{Z}$	Second time derivatives of X, Y, Z
X_m, Y_m, Z_m	Local reference system of the vehicle (Figure 2)
X_r, Y_r, Z_r	Coordinate of the radar in the earth-fixed X, Y, Z system
X_l, Y_l, Z_l	Radar coordinate system (Figure 1)
β	Ballistic coefficient
γ	Velocity aspect angle defined on page 11
δ	Vehicle re-entry angle
θ	Longitude of the vehicle
θ_r	Longitude of the radar
μ	A constant defined on page 4
ξ	Angle between $\vec{R}$ and $\vec{R}_o$ used in computing ground range
ρ	Atmospheric density

LIST OF SYMBOLS (cont.)

ϕ	Latitude of the vehicle
ϕ_r	Latitude of the radar
ψ	Bearing angle of the vehicle
ω	Earth's rotation rate

NOTE: With the exception of ground range all distances are in feet. Ground range is in nautical miles. β has units of lb/ft^2 and ρ is in $slugs/ft^2$

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INTRODUCTION

This trajectory program is intended for use when the vehicle can be considered as a point mass under the influence of gravity, atmospheric drag force, Coriolis force and centrifugal force. The input parameters have been chosen to be as few in number as possible and, at the same time, the ones most often used. For instance, the initial position of the vehicle is determined by its longitude as measured from Greenwich, its latitude above or below the equator and its height above the earth's surface. The position of the vehicle in a radar system is calculated from a knowledge of the longitude and latitude of the radar site.

The equations of motion, coordinate transformations and auxiliary computations are contained in the main body of the report. Appendix A contains the numerical integration procedure used to solve the equations of motion. Appendix B contains a list of the Fortran symbols and corresponding mathematical symbols, a complete listing of the Fortran statements and a flow diagram of the program. The input-output quantities are also defined in Appendix B.

The author would like to thank Mr. Thomas J. Kroupa III for his assistance in programming the equations for an IBM 1410 computer.

EQUATIONS OF MOTION

The trajectory of the vehicle is referenced to a right-handed rectangular co-ordinate system, x, y, z, rigidly connected to the rotating earth and with the origin at the earth's center. The z-axis is along the earth's polar axis and the xy plane is in the plane of the equator with the x-axis located at the meridian of Greenwich. (See Figure 1).

The forces acting on the vehicle in this system are gravitational, air resistance, Coriolis, and centrifugal. With the assumption that the force due to air resistance varies as $-kV^2$, the equations of motion along each of the co-ordinate axes are: (Reference 1)

$$\ddot{x} = \frac{-\mu^2 x}{(x^2 + y^2 + z^2)^{3/2}} - k\dot{x}(\dot{x}^2 + \dot{y}^2 + \dot{z}^2)^{\frac{1}{2}} + 2\dot{y}\omega + \omega^2 x \qquad (1)$$

$$\ddot{y} = \frac{-\mu^2 y}{(x^2 + y^2 + z^2)^{3/2}} - k\dot{y}(\dot{x}^2 + \dot{y}^2 + \dot{z}^2)^{\frac{1}{2}} - 2\dot{x}\omega + \omega^2 y \qquad (2)$$

$$\ddot{z} = \frac{-\mu^2 z}{(x^2 + y^2 + z^2)^{3/2}} - k\dot{z}(\dot{x}^2 + \dot{y}^2 + \dot{z}^2)^{\frac{1}{2}} \qquad (3)$$

where $\ddot{x}, \ddot{y}, \ddot{z} = \frac{d^2x}{dt^2}, \frac{d^2y}{dt^2}, \frac{d^2z}{dt^2}$ respectively, and

$\dot{x}, \dot{y}, \dot{z} = \frac{dx}{dt}, \frac{dy}{dt}, \frac{dz}{dt}$ respectively.

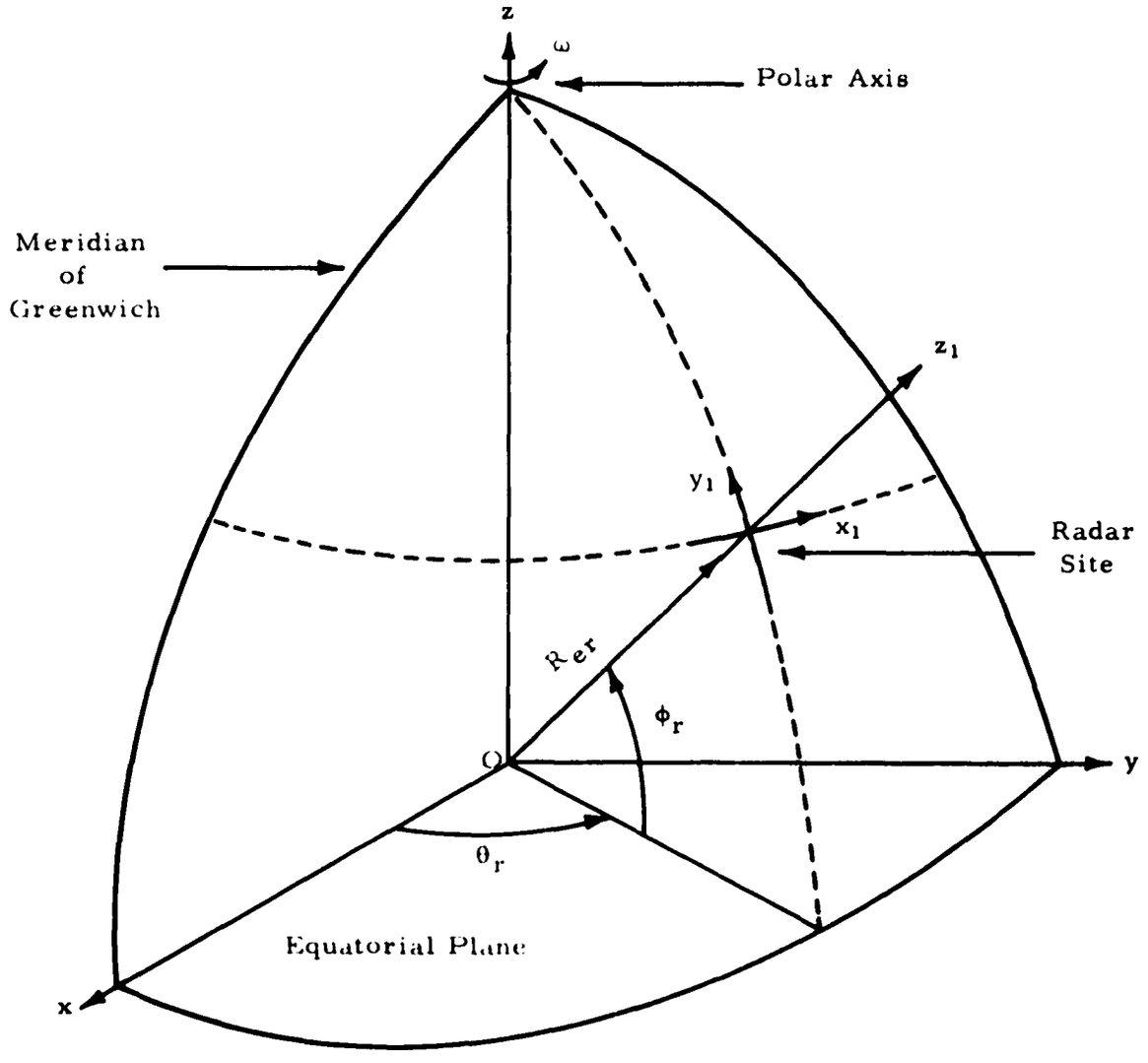

Figure 1

Relationship Between the Earth-Fixed x, y, z System
and the Radar x_1, y_1, z_1 System

$\mu^2 = g_o R_e^2$

g_o = acceleration of gravity at sea-level

R_e = mean radius of the earth

$k = \frac{1}{2} g_o \rho/\beta$

ρ = atmospheric density
ρ is computed as a function of altitude by a subroutine based on the ARDC Model Atmosphere, 1959 (Ref. 4)

$\beta = W/C_D A$, the ballistic coefficient

W = the weight of the vehicle

C_D = drag coefficient

A = reference area

ω = earth's rotation rate

Equations (1), (2), and (3) were numerically integrated by the fourth order method of Runge-Kutta as outlined in Appendix A. To start the integration procedure, a point in the 7-dimensional configuration space $t\ x\ y\ z\ \dot{x}\ \dot{y}\ \dot{z}$ must be known. This point is determined from the usual earth referenced trajectory parameters (speed, altitude, latitude, longitude, bearing angle and re-entry angle) by the following transformations. (See Figure 2)

$$\begin{aligned} x &= [R_e(\phi) + H] \cos\phi \cos\theta \\ y &= [R_e(\phi) + H] \cos\phi \sin\theta \\ z &= [R_e(\phi) + H] \sin\theta \end{aligned} \qquad (4)$$

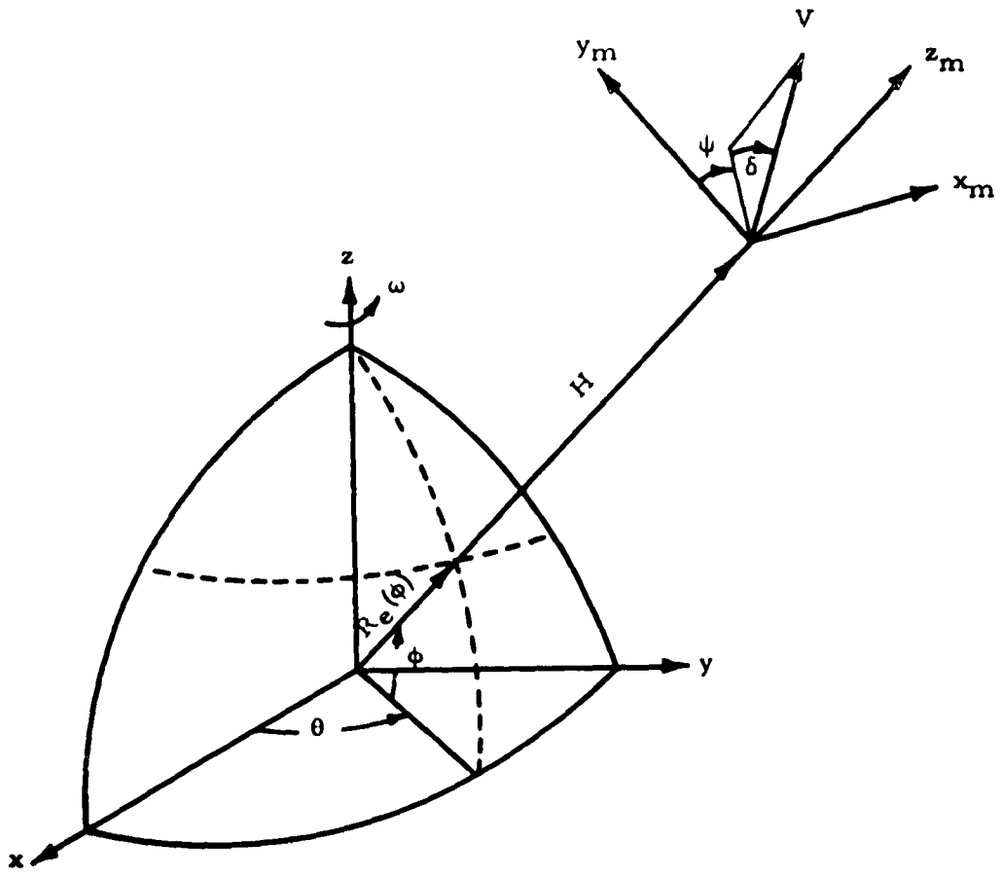

Figure 2

Relationship Between the Earth-Fixed x, y, z System and the Vehicle Local Reference System x_m, y_m, z_m

$$\begin{vmatrix} \dot{x} \\ \dot{y} \\ \dot{z} \end{vmatrix} = \begin{vmatrix} -\sin\theta & -\sin\phi\cos\theta & \cos\phi\cos\theta \\ \cos\theta & -\sin\phi\sin\theta & \cos\phi\sin\theta \\ 0 & \cos\phi & \sin\phi \end{vmatrix} \begin{vmatrix} V_{x_m} \\ V_{y_m} \\ V_{z_m} \end{vmatrix} \quad (5)$$

where $V_{x_m} = V \cos\delta \sin\psi$

$V_{y_m} = V \cos\delta \cos\psi$

$V_{z_m} = V \sin\psi$

θ = longitude of the vehicle

ϕ = latitude of the vehicle

δ = re-entry angle (positive upward - see Figure 2)

ψ = bearing angle (positive clockwise from north - see Figure 2)

V_{x_m}, V_{y_m}, V_{z_m} = the components of V in the vehicle's local reference system - see Figure 2

H = altitude above the earth's surface

$R_e(\phi)$ = radius of the earth at latitude ϕ

$R_e(\phi) = 20855967(1 - .00672267 \cos^2\phi)^{-\frac{1}{2}}$

Equations (4) and (5) are integral parts of the computer program.

The vehicle's position at any time during the integration is given by:

$$H = (x^2 + y^2 + z^2)^{\frac{1}{2}} - R_e(\phi) \quad (6)$$

$$\phi = \tan^{-1} \frac{z}{(x^2 + y^2)^{\frac{1}{2}}} \tag{7}$$

where if z is positive, $0 < \phi \leq 90°$ (north latitude), and if z is negative, $-90° \leq \phi < 0$ (south latitude).

$$\theta = \tan^{-1} \frac{y}{x} \tag{8}$$

To remove the ambiguity from θ, east longitudes were chosen to be positive, and west longitudes negative. The value of θ is determined from Subroutine QUAD by the following scheme:

y	x	θ
+	+	$0° \leq \theta \leq 90°$
+	-	$90° \leq \theta \leq 180°$
-	+	$-90° \leq \theta \leq 0°$
-	-	$-180° \leq \theta \leq -90°$

Ground range (S) is defined to be the distance traveled from the initial point along the earth's surface and is computed in increments as follows:

Let $\vec{R} = \vec{i}x + \vec{j}y + \vec{k}z$ at time t

and $\vec{R}_o = \vec{i}x_o + \vec{j}y_o + \vec{k}z_o$ at time $t - \Delta t$.

Then:

$$|\vec{R}_o \times \vec{R}| = |\vec{R}_o| \, |\vec{R}| \sin \xi$$

where ξ is the angle between the two vectors. Since the computation

interval Δt is very small, $\vec{R}$ will differ very little from $\vec{R}_o$, and and $\sin \xi \cong \xi$.

Thus,

$$\xi \cong |\vec{R}_o \times \vec{R}| \Big/ |\vec{R}_o|\ |\vec{R}| \ , \text{ and}$$

$$\Delta S \cong R_e \xi \quad . \tag{9}$$

ΔS is summed at the end of each computation interval to give S.

The re-entry angle, δ , is given at any time by:

$$\vec{R} \cdot \vec{V} = |\vec{R}||\vec{V}| \cos (\pi/2 - \delta)$$

$$= |\vec{R}||\vec{V}| \sin \delta$$

$$\delta = \sin^{-1} \vec{R} \cdot \vec{V} \Big/ |\vec{R}||\vec{V}| \tag{10}$$

where:

$$\vec{R} = \vec{i}x + \vec{j}y + \vec{k}z$$

and,

$$\vec{V} = \vec{i}x + \vec{j}y + \vec{k}z \quad .$$

δ is defined to be positive when above the local horizontal and negative when below. (See Figure 2).

TRAJECTORY PARAMETERS IN A RADAR REFERENCE SYSTEM

The co-ordinate system x_l, y_l, z_l with origin O_l at the radar is defined as follows: (See Figure 1)

x_l, y_l, z_l are co-ordinate axes with origin O_l at the surface of the earth, the x_l, y_l plane is perpendicular to a radius vector drawn from the center of the earth and the z_l axis is along the radius vector, the positive direction for x_l and y_l are taken to be due east and due north respectively.

For a station at latitude ϕ_r and longitude θ_r, the co-ordinates of O_l in the earth-fixed x, y, z system are:

$$\begin{aligned} x_r &= R_{er} \cos \phi_r \cos \theta_r \\ y_r &= R_{er} \cos \phi_r \sin \theta_r \\ z_r &= R_{er} \sin \phi_r \end{aligned} \tag{11}$$

where R_{er} is the value of $R_e(\phi)$ at ϕ_r.

Using the standard equation for translation and rotation of co-ordinate axes, the following relationship between the two systems is obtained. From x, y, z to x_l, y_l, z_l:

$$\begin{vmatrix} x_l \\ y_l \\ z_l \end{vmatrix} = \begin{vmatrix} -\sin \theta_r & \cos \theta_r & 0 \\ -\sin \phi_r \cos \theta_r & -\sin \phi_r \sin \theta_r & \cos \phi_r \\ \cos \phi_r \cos \theta_r & \cos \phi_r \sin \theta_r & \sin \phi_r \end{vmatrix} \begin{vmatrix} x - x_r \\ y - y_r \\ z - z_r \end{vmatrix} \tag{12}$$

from x_1, y_1, z_1 to x, y, z:

$$\begin{vmatrix} x \\ y \\ z \end{vmatrix} = \begin{vmatrix} -\sin\theta_r & -\sin\phi_r \cos\theta_r & \cos\phi_r \cos\theta_r \\ \cos\theta_r & -\sin\phi_r \sin\theta_r & \cos\phi_r \sin\theta_r \\ 0 & \cos\phi_r & \sin\phi_r \end{vmatrix} \begin{vmatrix} x_1 \\ y_1 \\ z_1 \end{vmatrix} + \begin{vmatrix} x_r \\ y_r \\ z_r \end{vmatrix} \quad (13)$$

In the computer program, these transformations are executed by the subroutines COOD and COODI. Thus, if one wishes to define the radar system in some other manner, only the subroutines will have to be changed.

After the vehicle's position has been transformed from the x, y, z system to the x_1, y_1, z_1 system, the slant range, azimuth angle, and elevation angle are computed as follows:

$$R_1 = (x_1^2 + y_1^2 + z_1^2)^{\frac{1}{2}} \quad (14)$$

$$E_l = \tan^{-1}\left[z_1 / (x_1^2 + y_1^2)^{\frac{1}{2}}\right] \quad (15)$$

$$A_z = \tan^{-1}\left(\frac{x_1}{y_1}\right) \quad (16)$$

E_l ranges from 0° to 90° and is positive if the vehicle is above the horizon.

A_z ranges from 0° to 360° and is measured positive clockwise from north.

R_1, E_l, and A_z are computed in subroutine RAE. The comments made about COOD and COODI also apply to RAE.

$\vec{R}_1$ can be expressed in the x, y, z system as:

$$\vec{R}_1 = \vec{i}\,(x - x_r) + \vec{j}\,(y - y_r) + \vec{k}\,(z - z_r) \quad ,$$

and the velocity vector in the same system is:

$$\vec{V} = \vec{i}\,x + \vec{j}\,y + \vec{k}\,z \quad .$$

Using these two equations, the velocity aspect angle and the range rate can be computed.

The velocity aspect angle (angle between the radar line of sight and the velocity vector) is given by:

$$\gamma = \cos^{-1} \frac{\vec{R}_1 \cdot \vec{V}}{|R_1|\,|V|} \tag{17}$$

The component of $|\vec{V}|$ along R_1 is the range-rate ($\dot{R}_1$). Thus,

$$\dot{R}_1 = |\vec{V}| \cos \gamma \tag{18}$$

Since $\dot{R}_1$ is negative when the vehicle is approaching the radar, γ is chosen to range from 0° to 180°. $\gamma = 0°$ when the vehicle is going directly away from the radar, and $\gamma = 180°$ when the vehicle is headed straight in.

CONCLUSIONS

The output of the computer program has been compared to actual radar data and found to be in good agreement. It is felt that the program will be useful for generating theoretical slowdown curves and for determining range, range rates, and look-angles from arbitrary radar locations.

APPENDIX A

NUMERICAL INTEGRATION OF THE EQUATIONS OF MOTION

The method described below in the classic fourth order procedure of Runge-Kutta (Reference 2). Only one point on the integral curves is needed to start the integration, and with the aid of high speed computing machines, any degree of accuracy can be achieved by choosing a sufficiently small increment of the independent variable.

Writing equations (1), (2), and (3) as:

$$\ddot{x} = f_1(t, x, y, z, \dot{x}, \dot{y}, \dot{z}) \tag{1A}$$

$$\ddot{y} = f_2(t, x, y, z, \dot{x}, \dot{y}, \dot{z}) \tag{2A}$$

$$\ddot{z} = f_3(t, x, y, z, \dot{x}, \dot{y}, \dot{z}) \tag{3A}$$

The integration proceeds as follows.

Let t take on an increment Δt; then $x, y, z, \dot{x}, \dot{y}$, and $\dot{z}$ receive increments K_1, K_2, K_3, K_4, K_5, and K_6 respectively.

$$K_1 = 1/6\,(k_{11} + 2k_{21} + 2k_{31} + k_{41}) \tag{4A}$$

$$K_2 = 1/6\,(k_{12} + 2k_{22} + 2k_{32} + k_{42}) \tag{5A}$$

$$K_3 = 1/6\,(k_{13} + 2k_{23} + 2k_{33} + k_{43}) \tag{6A}$$

$$K_4 = 1/6\,(k_{14} + 2k_{24} + 2k_{34} + k_{44}) \tag{7A}$$

$$K_5 = 1/6\,(k_{15} + 2k_{25} + 2k_{35} + k_{45}) \tag{8A}$$

$$K_6 = 1/6\,(k_{16} + 2k_{26} + 2k_{36} + k_{46}) \tag{9A}$$

$$k_{11} = \dot{x}\,\Delta t \tag{10A}$$

$$k_{12} = \dot{y}\,\Delta t \tag{11A}$$

$$k_{13} = \dot{z}\,\Delta t \tag{12A}$$

$$k_{14} = f_1\,(t, x, y, z, \dot{x}, \dot{y}, \dot{z})\,\Delta t \tag{13A}$$

$$k_{15} = f_2(t, x, y, z, \dot{x}, \dot{y}, \dot{z})\,\Delta t \tag{14A}$$

$$k_{16} = f_3\,(t, x, y, z, \dot{x}, \dot{y}, \dot{z})\,\Delta t \tag{15A}$$

$$k_{21} = (\dot{x} + \tfrac{1}{2}k_{14})\,\Delta t \tag{16A}$$

$$k_{22} = (\dot{y} + \tfrac{1}{2}k_{15})\,\Delta t \tag{17A}$$

$$k_{23} = (\dot{z} + \tfrac{1}{2}k_{16})\,\Delta t \tag{18A}$$

$$k_{24} = f_1\,(t + \tfrac{1}{2}\Delta t,\ x + \tfrac{1}{2}k_{11},\ y + \tfrac{1}{2}k_{12},\ z + \tfrac{1}{2}k_{13},\ \dot{x} + \tfrac{1}{2}k_{14},\ \dot{y} + \tfrac{1}{2}k_{15},\ \dot{z} + \tfrac{1}{2}k_{16})\,\Delta t \tag{19A}$$

$$k_{25} = f_2 \left(t + \tfrac{1}{2}\Delta t,\ x + \tfrac{1}{2}k_{11},\ y + \tfrac{1}{2}k_{12},\ z + \tfrac{1}{2}k_{13},\ \dot{x} + \tfrac{1}{2}k_{14},\ \dot{y} + \tfrac{1}{2}k_{15},\ \dot{z} + \tfrac{1}{2}k_{16},\right) \Delta t \tag{20A}$$

$$k_{26} = f_3 \left(t + \tfrac{1}{2}\Delta t,\ x + \tfrac{1}{2}k_{11},\ y + \tfrac{1}{2}k_{12},\ z + \tfrac{1}{2}k_{13},\ \dot{x} + \tfrac{1}{2}k_{14},\ \dot{y} + \tfrac{1}{2}k_{15},\ \dot{z} + \tfrac{1}{2}k_{16},\right) \Delta t \tag{21A}$$

$$k_{31} = \left(\dot{x} + \tfrac{1}{2}k_{24}\right) \Delta t \tag{22A}$$

$$k_{32} = \left(\dot{y} + \tfrac{1}{2}k_{25}\right) \Delta t \tag{23A}$$

$$k_{33} = \left(\dot{z} + \tfrac{1}{2}k_{26}\right) \Delta t \tag{24A}$$

$$k_{34} = f_1 \left(t + \tfrac{1}{2}\Delta t,\ x + \tfrac{1}{2}k_{21},\ y + \tfrac{1}{2}k_{22},\ z + \tfrac{1}{2}k_{23},\ \dot{x} + \tfrac{1}{2}k_{24},\ \dot{y} + \tfrac{1}{2}k_{25},\ \dot{z} + \tfrac{1}{2}k_{26}\right) \Delta t \tag{25A}$$

$$k_{35} = f_2 \left(t + \tfrac{1}{2}\Delta t,\ x + \tfrac{1}{2}k_{21},\ y + \tfrac{1}{2}k_{22},\ z + \tfrac{1}{2}k_{23},\ \dot{x} + \tfrac{1}{2}k_{24},\ \dot{y} + \tfrac{1}{2}k_{25},\ \dot{z} + \tfrac{1}{2}k_{26}\right) \Delta t \tag{26A}$$

$$k_{35} = f_3 \left(t + \tfrac{1}{2}\Delta t,\ x + \tfrac{1}{2}k_{21},\ y + \tfrac{1}{2}k_{22},\ z + \tfrac{1}{2}k_{23},\ \dot{x} + \tfrac{1}{2}k_{24},\ \dot{y} + \tfrac{1}{2}k_{25},\ \dot{z} + \tfrac{1}{2}k_{26}\right) \Delta t \tag{27A}$$

$$k_{41} = \left(\dot{x} + k_{34}\right)\Delta t \tag{28A}$$

$$k_{42} = \left(\dot{y} + k_{35}\right) \Delta t \tag{29A}$$

$$k_{43} = \left(\dot{z} + k_{36}\right) \Delta t \tag{30A}$$

$$k_{44} = f_1 \left(t + \Delta t,\ x + k_{31},\ y + k_{32},\ z + k_{33},\ \dot{x} + k_{34},\ \dot{y} + k_{35},\ \dot{z} + k_{36}\right) \Delta t \tag{31A}$$

$$k_{45} = f_2 (t + \Delta t,\ x + k_{31},\ y + k_{32},\ z + k_{33},\ \dot{x} + k_{34},\ \dot{y} + k_{35},\ z + k_{36})\ \Delta t \tag{32A}$$

$$k_{46} = f_3 (t + \Delta t,\ x + k_{31}, y + k_{32},\ z + k_{33},\ \dot{x} + k_{34},\ \dot{y} + k_{35}\ \dot{z} + k_{36})\ \Delta t \tag{33A}$$

On the first pass through equations 10A to 33A, the variables t, x, y, z, $\dot{x}$, $\dot{y}$, $\dot{z}$ will have their initial values. After equation 33A has been executed, t is incremented by Δt, x by K_1, y by K_2, z by K_3, $\dot{x}$ by K_4, $\dot{y}$ by K_5, and $\dot{z}$ by K_6 and the procedure beginning at 10A is repeated.

During portions of the trajectory where the acceleration is small Δt may be chosen fairly large (around $\frac{1}{4}$ sec), but when the acceleration is large, Δt must be small (around 1/100 sec).

Equations f_1, f_2, and f_3 are evaluated in the program by subroutine FU123.

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APPENDIX B

List of FORTRAN Symbols

Inputs to the Program

Output of the Program

Sample Print-Out

List of FORTRAN Statements

Flow Diagram

LIST OF FORTRAN SYMBOLS

FORTRAN Symbols	Mathematical Symbols
AK1, AK2, AK3	K_1, K_2, K_3
AK4, AK5, AK6	K_4, K_5, K_6
AK11, AK12, AK13	k_{11}, k_{12}, k_{13}
AK14, AK15, AK16	k_{14}, k_{15}, k_{16}
AK21, AK22, AK23	k_{21}, k_{22}, k_{23}
AK24, AK25, AK26	k_{24}, k_{25}, k_{26}
AK31, AK32, AK33	k_{31}, k_{32}, k_{33}
AK34, AK35, AK36	k_{34}, k_{35}, k_{36}
AK41, AK42, AK43	k_{41}, k_{42}, k_{43}
AK44, AK45, AK46	k_{44}, k_{45}, k_{46}
ALA	ϕ_r
ALO	θ_r
AT	$\ddot{x}^2 + \ddot{y}^2 + \ddot{z}^2$
AZ	A_z
BC	β
BETA	ψ
DEL	δ
EL	Ξ_l
DT	Δt
Gamma	γ

FORTRAN Symbols	Mathematical Symbols
H	H
PHI	ϕ
RAN	S
RER	$R_e(\phi)$ at $\phi = \phi_r$
RO	ρ
RV	$H + R_e(\phi)$
R1	R_1
RR1	$\dot{R}_1$
T	t
THETA	θ
V	V
VT	$(\dot{x}^2 + \dot{y}^2 + \dot{z}^2)^{\frac{1}{2}}$
VX, VY, VZ	V_{x_m}, V_{y_m}, V_{z_m}
X, Y, Z	X, Y, Z
XD, YD, ZD	$\dot{X}, \dot{Y}, \dot{Z}$
XDD, YDD, ZDD	$\ddot{X}, \ddot{Y}, \ddot{Z}$
XR, YR, ZR	X_r, Y_r, Z_r
X1, Y1, Z1	X_1, Y_1, Z_1

INPUTS TO THE PROGRAM

The initial conditions are read into the computer from four input cards containing the following information:

Card #1 BC, DEL, BETA, V

BC = the ballistic coefficient (W/C_DA) in lbs/ft^2

DEL = the re-entry angle (negative when re-entering) in degrees

BETA = the velocity bearing angle (positive clockwise from north) in degrees

V = magnitude of the velocity in ft/sec

Card #2 THETA, PHI, H

THETA = longitude of the vehicle in degrees. If the longitude is given as θ degrees west of Greenwich, change to $360° - \theta$.

PHI = latitude of vehicle in degrees - input as positive when above the Equator and negative when below

H = altitude above the earth's surface in feet

Card #3 ALO, ALA

ALO = longitude of radar site (input in the same manner as THETA)

ALA = latitude of the radar site (input in the same manner as PHI)

Card #4 DT, HEND, N

DT = increment of the independent variable time

HEND = altitude at which it is desired that the computation be halted

N = print rate - controls the number of times through the integration loop before printing. For instance, if DT is 1/10 sec and trajectory data is desired at 1 sec intervals, then N should be read in as 10.

OUTPUT OF THE PROGRAM

The quantities shown on the sample print-out are defined as follows:

TIME = Elapsed time in seconds from initial point

LATITUDE = Latitude of the vehicle in degrees (positive when north of the equator)

LONGITUDE = Longitude of the vehicle in degrees (positive when east of Greenwich)

ALTITUDE = Height above the earth's surface in feet

TOTAL ACCELERATION = Absolute value of the acceleration (ft/sec^2) in the earth-fixed reference system

TOTAL VELOCITY = Absolute value of the velocity (ft/sec) in the earth-fixed reference system

GROUND RANGE = Distance traveled over the earth's surface from the initial point (nautical miles)

RE-ENTRY ANGLE = Angle between the velocity vector and the local horizontal (degrees)

X RADAR
Y RADAR = Co-ordinates of the vehicle in the radar reference
Z RADAR system (feet)

AZIMUTH ANGLE
ELEVATION ANGLE = Radar look-angles in degrees (azimuth is no. of degrees clockwise from north, and elevation is no. of degrees above the horizon).

GAMMA = Angle between the radar line-of-sight and the velocity vector (degrees)

SLANT RANGE = Distance from the radar to the vehicle (feet)

RANGE RATE = Rate of change of the slant range (ft/sec)

TIME	0.16267E 05	LATITUDE	0.14475E-01	AZIMUTH ANGLE	0.89968E 02
ALTITUDE	0.66042E 07	LONGITUDE	-0.42695E 02	ELEVATION ANGLE	0.33062E 02
TOTAL ACCELERATION	0.15347E 02	RE-ENTRY ANGLE	0.10977E 02	GAMMA	0.39456E 02
X RADAR	0.82189E 07	Y RADAR	0.69542E 04	Z RADAR	0.53488E 07
TOTAL VELOCITY	0.19945E 05	SLANT RANGE	0.98061E 07		
GROUND RANGE	0.45807E 05	RANGE RATE	0.15402E 05		

TIME	0.16268E 05	LATITUDE	0.14470E-01	AZIMUTH ANGLE	0.89969E 02
ALTITUDE	0.66080E 07	LONGITUDE	-0.42654E 02	ELEVATION ANGLE	0.32988E 02
TOTAL ACCELERATION	0.15343E 02	RE-ENTRY ANGLE	0.10975E 02	GAMMA	0.39425E 02
X RADAR	0.82387E 07	Y RADAR	0.69525E 04	Z RADAR	0.53465E 07
TOTAL VELOCITY	0.19942E 05	SLANT RANGE	0.98215E 07		
GROUND RANGE	0.45809E 05	RANGE RATE	0.15406E 05		

TIME	0.16269E 05	LATITUDE	0.14465E-01	AZIMUTH ANGLE	0.89969E 02
ALTITUDE	0.66118E 07	LONGITUDE	-0.42614E 02	ELEVATION ANGLE	0.32914E 02
TOTAL ACCELERATION	0.15338E 02	RE-ENTRY ANGLE	0.10973E 02	GAMMA	0.39394E 02
X RADAR	0.82585E 07	Y RADAR	0.69509E 04	Z RADAR	0.53443E 07
TOTAL VELOCITY	0.19938E 05	SLANT RANGE	0.98369E 07		
GROUND RANGE	0.45812E 05	RANGE RATE	0.15410E 05		

TIME	0.16270E 05	LATITUDE	0.14459E-01	AZIMUTH ANGLE	0.89969E 02
ALTITUDE	0.66156E 07	LONGITUDE	-0.42573E 02	ELEVATION ANGLE	0.32841E 02
TOTAL ACCELERATION	0.15333E 02	RE-ENTRY ANGLE	0.10970E 02	GAMMA	0.39363E 02
X RADAR	0.82783E 07	Y RADAR	0.69493E 04	Z RADAR	0.53420E 07
TOTAL VELOCITY	0.19935E 05	SLANT RANGE	0.98523E 07		
GROUND RANGE	0.45814E 05	RANGE RATE	0.15414E 05		

TIME	0.16271E 05	LATITUDE	0.14454E-01	AZIMUTH ANGLE	0.89969E 02
ALTITUDE	0.66194E 07	LONGITUDE	-0.42532E 02	ELEVATION ANGLE	0.32767E 02
TOTAL ACCELERATION	0.15329E 02	RE-ENTRY ANGLE	0.10968E 02	GAMMA	0.39333E 02
X RADAR	0.82981E 07	Y RADAR	0.69476E 04	Z RADAR	0.53398E 07
TOTAL VELOCITY	0.19931E 05	SLANT RANGE	0.98677E 07		
GROUND RANGE	0.45817E 05	RANGE RATE	0.15418E 05		

TIME	0.16272E 05	LATITUDE	0.14449E-01	AZIMUTH ANGLE	0.89969E 02
ALTITUDE	0.66232E 07	LONGITUDE	-0.42491E 02	ELEVATION ANGLE	0.32694E 02
TOTAL ACCELERATION	0.15324E 02	RE-ENTRY ANGLE	0.10966E 02	GAMMA	0.39302E 02
X RADAR	0.83179E 07	Y RADAR	0.69460E 04	Z RADAR	0.53375E 07
TOTAL VELOCITY	0.19928E 05	SLANT RANGE	0.98831E 07		
GROUND RANGE	0.45819E 05	RANGE RATE	0.15422E 05		

LIST OF FORTRAN PROGRAM

MAIN PROGRAM

```
FORTRAN   RUN
          BCP
 LINE # C
 IPAGE # 1
 REAC 100,BC,DEL,BETA,V
 REAC 100,THETA,PHI,H
 REAC 10C,ALO,ALA
 REAC 101,DT,HEND,N
 PRINT 1C7
 PRINT 1C6
 PRINT 1C2,H,V ,DEL.BC,BETA,PHI,THETA
 RAN#O.
 T#O.
 M#O
 DEL#DEL*.01745
 BETA#BETA*.01745
 THETA#THETA*.01745
 PHI#PHI*.01745
 ALO#ALO*.01745
 ALA#ALA*.01745
```

LIST OF FORTRAN PROGRAM

MAIN PROGRAM

```
      RV#H&20855967./SQRTF%1.-.00672267*COSF%PHI□**2□
      RER#20855967./SQRTF%1.-.00672267*COSF%ALA□**2□
      XR#RER*COSF%ALA□*COSF%ALO□
      YR#RER*COSF%ALA□*SINF%ALO□
      ZR#RER*SINF%ALA□
      VX#V*COSF%CEL□*SINF%BETA□
      VY#V*COSF%CEL□*COSF%BETA□
      VZ#V*SINF%CEL□
      CALL COCO%XD ,YD ,ZD ,VX,VY,VZ,THETA,PHI□
      X#RV*COSF%PHI□*COSF%THETA□
      Y#RV*COSF%PHI□*SINF%THETA□
      Z#RV*SINF%PHI□
    1 XO#X
      YO#Y
      ZO#Z
      RVO#RV
      CALL ALT%H,RO,A,B,C,D,E,F,G□
      RC#RO/32.174
      AK11#XD*DT
      AK12#YD*DT
```

LIST OF FORTRAN PROGRAM

MAIN PROGRAM

```
 AK13#ZD*DT
 CALL FU123%AK14,AK15,AK16,X,Y,Z,XD,YD,ZD,RO,BC,DT¤
 AK21#%XD&.5*AK14¤*DT
 AK22#%YD&.5*AK15¤*DT
 AK23#%ZD&.5*AK16¤*DT
OCALL FU123%AK24,AK25,AK26,X&.5*AK11,Y&.5*AK12,Z&.5*AK13,XD&.5*AK14
1,YD&.5*AK15,ZD&.5*AK16,RO,BC,DT¤
 AK31#%XC&.5*AK24¤*DT
 AK32#%YD&.5*AK25¤*DT
 AK33#%ZD&.5*AK26¤*DT
OCALL FU123%AK34,AK35,AK36,X&.5*AK21,Y&.5*AK22,Z&.5*AK23,XD&.5*AK24
1,YD&.5*AK25,ZD&.5*AK26,RO,BC,DT¤
 AK41#%XD&AK34¤*DT
 AK42#%YC&AK35¤*DT
 AK43#%ZC&AK36¤*DT
OCALL FU123%AK44,AK45,AK46,X&AK31,Y&AK32,Z&AK33,XD&AK34,YD&AK35,ZD&
1AK36,RO,BC,DT¤
 AK1#%AK11&2.*AK21&2.*AK31&AK41¤/6.
 AK2#%AK12&2.*AK22&2.*AK32&AK42¤/6.
 AK3#%AK13&2.*AK23&2.*AK33&AK43¤/6.
```

LIST OF FORTRAN PROGRAM

MAIN PROGRAM

```
      AK4#%AK14&2.*AK24&2.*AK34&AK44□/6.
      AK5#%AK15&2.*AK25&2.*AK35&AK45□/6.
      AK6#%AK16&2.*AK26&2.*AK36&AK46□/6.
      X#X&AK1
      Y#Y&AK2
      Z#Z&AK3
      XC#XC&AK4
      YC#YD&AK5
      ZC#ZC&AK6
      RV#SQRTF%X*X&Y*Y&Z*Z□
      H#RV-20855967./SQRTF%1.-.00672267*%X*X&Y*Y□/RV**2□
      B1#%Z*YC-Y*ZO□**2&%X*ZO-Z*XC□**2&%Y*XO-X*YO□**2
      C#SQRTF%B1□/%RV*RVO□
      RAN#C*2.078505E&07/6076.1&RAN
      T#T&DT
      M#M&1
      IF%N-M□2,2,1
    2 CALL QUAD%THETA,PHI,X,Y,Z□
      M#0
      VT#SQRTF%XC*XD&Y*YD&ZC*ZD□
```

LIST OF FORTRAN PROGRAM

MAIN PROGRAM

```
      CALL EU123%XDD,YDD,ZDD,X,Y,Z,XO,YO,ZO,RO,BC,L.¤
      AT#SQRTF%XDD*XDD&YDD*YDD&ZDD*ZDD¤
      D#%X*XD&Y*YD&Z*ZD¤/%RV*VT¤
      D1#SQRTF%1.-D*D¤
      DEL#ATANF%D/D1¤/.0174533
      CALL COCD!%X1,Y1,Z1,X-XR,Y-YR,Z-ZR,ALO,ALA¤
      CALL RAE%R1,AZ,EL,X1,Y1,Z1¤
      RR1#%%X-XR¤*XD&%Y-YR¤*YD&%Z-ZR¤*ZD¤/R1
      CGAMA#RR1/VT
      SGAMA#SQRTF%1.-CGAMA**2¤
      IF%RR1¤12,12,13
   13 GAMA#ATANF%SGAMA/CGAMA¤/.01745
      GO TO 15
   12 GAMA#180.-ABSF%ATANF%SGAMA/CGAMA¤/.01745¤
   15 CONTINUE
      PRINT 200,T,PHI,AZ
      PRINT 201,H,THETA,EL
      PRINT 202,AT,DEL,GAMA
      PRINT 1111,X1,Y1,Z1
      PRINT 203,VT,R1
```

LIST OF FORTRAN PROGRAM

MAIN PROGRAM

```
      PRINT 2C4,RAN,RR1
      LINE # LINE & 1
      IF%LINE - 6¤1234,1423,1234
1423  LINE # C
      IPAGE #   IPAGE & 1
      PRINT 1532,IPAGE
1532  FORMAT%1H1,75X,4HPAGE,I5¤
1234  CONTINUE
      IF%H-HEND¤4,4,1
   4  PRINT 5CO
      STOP
 100  FORMAT%8E15.8¤
 101  FORMAT%2E15.8,I3
 107  FORMAT%1H1,57X,18HINITIAL CONDITIONS¤
 106¤FORMAT%1HK,25X,3HALT,5X,8HVELOCITY,3X,10HRE-ENT ANG,7X,6HBAL CO,5X
     1,8HBEAR ANG,9X,4HLATO,9X,4HLONO¤
 102  FORMAT%1H ,17X,10E13.5¤
 200¤FORMAT%///,16X,4HTIME,18X,E12.5,6X,8HLATITUDE,9X,E12.5,5X,13HAZIMU
     1TH ANGLE,5X,E12.5¤
 201¤FORMAT%1H ,15X,8HALTITUDE,14X,E12.5,6X,9HLONGITUDE,8X,E12.5,5X,15H
```

LIST OF FORTRAN PROGRAM

MAIN PROGRAM

```
     1ELEVATION ANGLE,3X,E12.5¤
  2020FORMAT%1H ,15X,18HTOTAL ACCELERATION,4X,E12.5,6X,14HRE-ENTRY ANGLE
     1,3X,E12.5,5X,5HGAMMA,13X,E12.5¤
  2030FORMAT%1H ,15X,14HTOTAL VELOCITY,8X,E12.5,6X,11HSLANT RANGE,6X,E12
     1.5¤
   204 FORMAT%1H ,15X,12HGROUND RANGE,10X,E12.5,6X,10HRANGE RATE,7X,E12.5
     1¤
 1111 FORMAT%1H ,15X,8HX RADAR ,14X,E12.5,6X,8HY RADAR ,9X,E12.5,5X,7HZ
     1RADAR,11X,E12.5¤
   500 FORMAT%1H1,60X,10HEND OF JOB /////////¤
       END
```

LIST OF FORTRAN PROGRAM

SUBROUTINE COOD

```
      BCP  COOD
SUBROUTINE COOD%A,B,C,D,E,F,O,P¤
A#-C*SINF%O¤-E*SINF%P¤*COSF%O¤&F*COSF%P¤*COSF%O¤
B#D*COSF%O¤-E*SINF%P¤*SINF%C¤&F*COSF%P¤*SINF%O¤
C#E*COSF%P¤&F*SINF%P¤
RETURN
END
```

LIST OF FORTRAN PROGRAM

SUBROUTINE QUAD

```
                BCP  QUAD
      SUBROUTINE QUAD%A,B,X,Y,Z0
      A#ATANF%Y/X0/.01745
      IF%Y01.2,2
    1 IF%X03,4,20
    3 A # -180. & A
      GO TO 20
    4 A#90.
      GO TO 20
    2 IF%X06,4,20
    6 A#180.&A
      GO TO 20
   20 B#ATANF%Z/%SQRTF%X*X&Y*Y000/.01745
      RETURN
      END
```

LIST OF FORTRAN PROGRAM

SUBROUTINE RAE

```
      BCP  RAE
      SUBROUTINE RAE%A,B,C,X1,Y1,Z1¤
      A#SQRTF%X1*X1 & Y1*Y1 & Z1*Z1¤
      C#ATANF%Z1/SQRTF%X1*X1 & Y1*Y1¤¤/.01745
      B#ATANF%X1/Y1¤/.01745
      IF%X1¤1,2,2
    1 IF%Y1¤3,4,5
    3 B#180.&B
      GO TO 15
    4 B#90.
      GO TO 15
    5 B#360.&B
      GO TO 15
    2 IF%Y1¤6,4,15
    6 B#180.&B
      GO TO 15
   15 RETURN
      END
```

LIST OF FORTRAN PROGRAM

SUBROUTINE COODI

```
      BCP  COODI
SUBROUTINE COODI%A,B,C,D,E,F,O,P¤
A#-C*SINF%C¤&E*COSF%O¤
B#-C*SINF%P¤*COSF%O¤-E*SINF%P¤*SINF%O¤&F*COSF%P¤
C#C*COSF%P¤*COSF%C¤&E*COSF%P¤*SINF%O¤&F*SINF%P¤
RETURN
END
```

LIST OF FORTRAN PROGRAM

SUBROUTINE FU123

```
      BCP  FU123
SUBROUTINE FU123%A,B,C,X,Y,Z,XD,YD,ZD,RO,BC,DT¤
C2#1.38999091E&16
U#.72918296E-04
C3#16.087*RC/BC
D1#%X*X&Y*Y&Z*Z¤**1.5
D2#SQRTF%XD*XD&YD*YD&ZD*ZD¤
A#%-C2*X/D1-C3*XD*D2&2.*U*YC&U*U*X¤*DT
B#%-C2*Y/D1-C3*YD*D2-2.*U*XC&U*U*Y¤*DT
C#%-C2*Z/D1-C3*ZD*D2¤*DT
RETURN
END
```

LIST OF FORTRAN PROGRAM

SUBROUTINE ALT

```
          BCP  ALT
      SUBROUTINE ALT%H,RO,PR,CF,FP,TEM,GRA,WM,SOS¤
      IF%H-40C00C.¤30,30,31
   31 RC#0.
      RETURN
   30 IF%H-40C00.¤1,2,3
    1 X#H/100C0.
      P3# -0.56721846E-03
      P2# -0.958C8049E-02
      P1# -0.37347339E&00
      P0#  0.19173574E&02
      T3#  0.12881606E&01
      T2# -0.60534827E&01
      T1# -0.28813737E&C2
      T0#  0.51772365E&C3
      GO TO 5C
    2 CONTINUE
    3 IF%H-8CC00.¤4,5,6
    4 CONTINUE
    5 X#%H-40000.¤/10000.
```

LIST OF FORTRAN PROGRAM

SUBROUTINE ALT

```
      P3#=0.39193446E-05
      P2# 0.25197324E-03
      P1#=0.47883423E&00
      P0# 0.17487069E&02
      T3# 0.
      T2# 0.
      T1# 0.
      T0# 0.38999000E&03
      GO TO 5C
    1 IF(H-16C000.)7,8,9
    7 X#=H-80C00.)/1000C.
      P3# -0.19423133E-03
      P2#  0.99676253E-02
      P1# -0.48090213E&00
      P0#  0.15575633E&02
      T3# -0.10962868E&00
      T2#  0.12311886E&01
      T1#  0.12416913E&02
      T0#  0.38925805E&03
      GO TO 5C
```

SUBROUTINE ALT

```
  8 CONTINUE
  9 IF(H-17500C.)10,11,12
 10 CONTINUE
 11 X=(H-16000C.)/1COCO.
    P3= C.0COO
    P2= 0.1680C000E-03
    P1=-0.36283040E+00
    PO= 0.12266563E+02
    T3= 0.
    T2= 0.
    T1= C.
    TC= 0.50879000E+03
    GO TO 5C
 12 IF(H-27C000.)13,14,15
 13 X=(H-17500C.)/10000.
    P3= -0.50571711E-03
    P2= -0.71458221E-02
    P1= -0.36369539E+00
    PO=  0.11723187E+02
    T3=  0.12190679E+CO
```

SUBROUTINE ALT

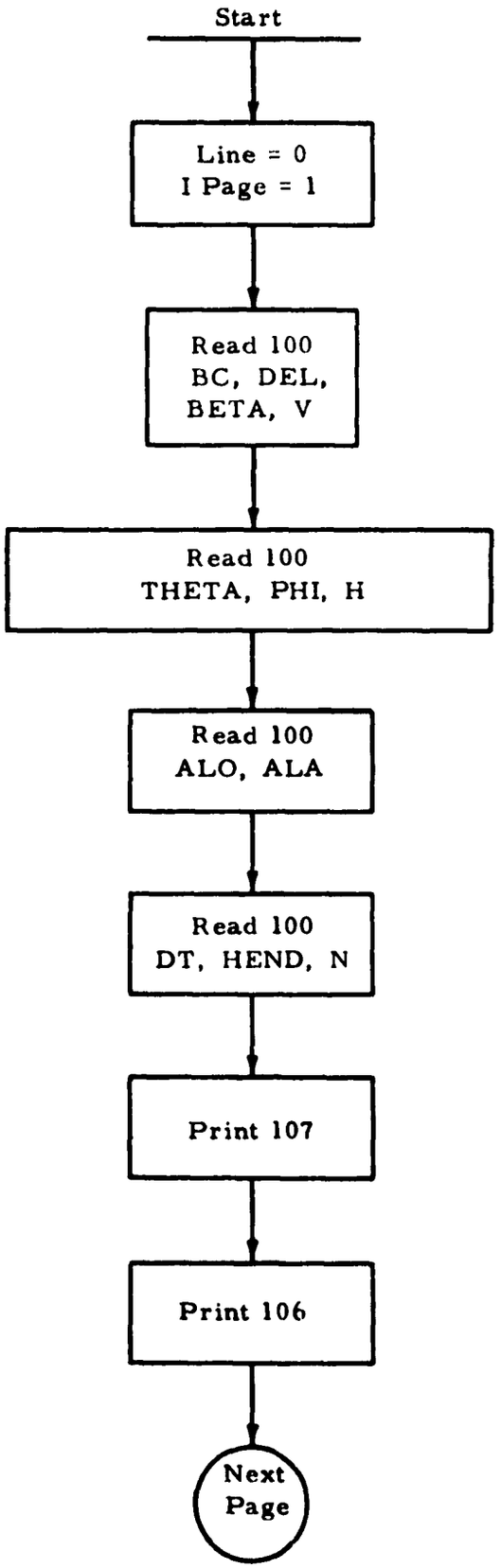
```
      T2# -0.13745684E&01
      T1# -0.20357798E&02
      T0#  0.50755152E&03
      GO TO 50
14 CONTINUE
15 IF%H-290000.¤16,17,18
16 CONTINUE
17 X#%H-270000.¤/10000.
      P3# 0.
      P2# 0.25095006E-03
      P1#-0.61251035E&00
      P0# 0.71906764E&01
      T3# 0.
      T2# 0.
      T1# 0.
      T0# 0.2982C000E&03
      GO TO 50
18 IF%H-35C00C.¤19,20,21
19 X#%H-300000.¤/10000.
      P3# -0.12243014E-03
```

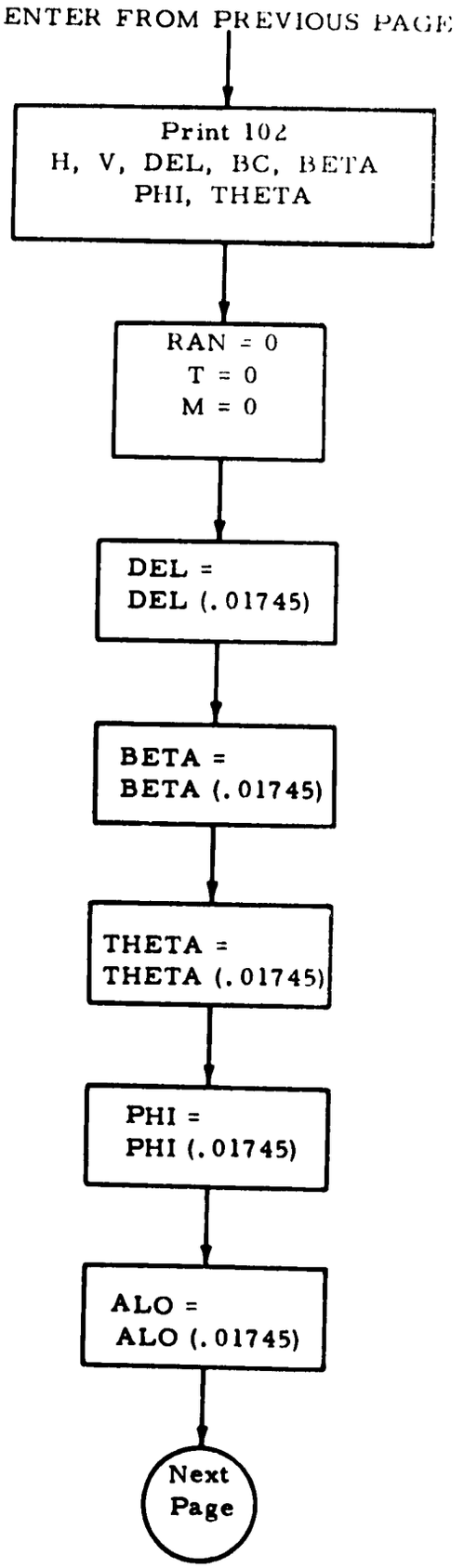
```
      P2#  0.16998582E-01
      P1# -0.63252519E&00
      P0#  0.59679151E&01
      T3# -0.55833219E&00
      T2#  0.61892744E&01
      T1#  0.66193506E&00
      T0#  0.29675713E&03
      GO TO 50
   20 CONTINUE
   21 X#%H-350000.¤/10000.
      P3# -0.26309606E-02
      P2#  0.43390852E-01
      P1# -0.43767363E&00
      P0#  0.27568534E&01
      T3# -0.10924334E&00
      T2#  0.84074361E&00
      T1#  0.10253056E&03
      T0#  0.40381341E&03
   50 IF%H-295000.¤22,22,23
   22 WM#28.966
```

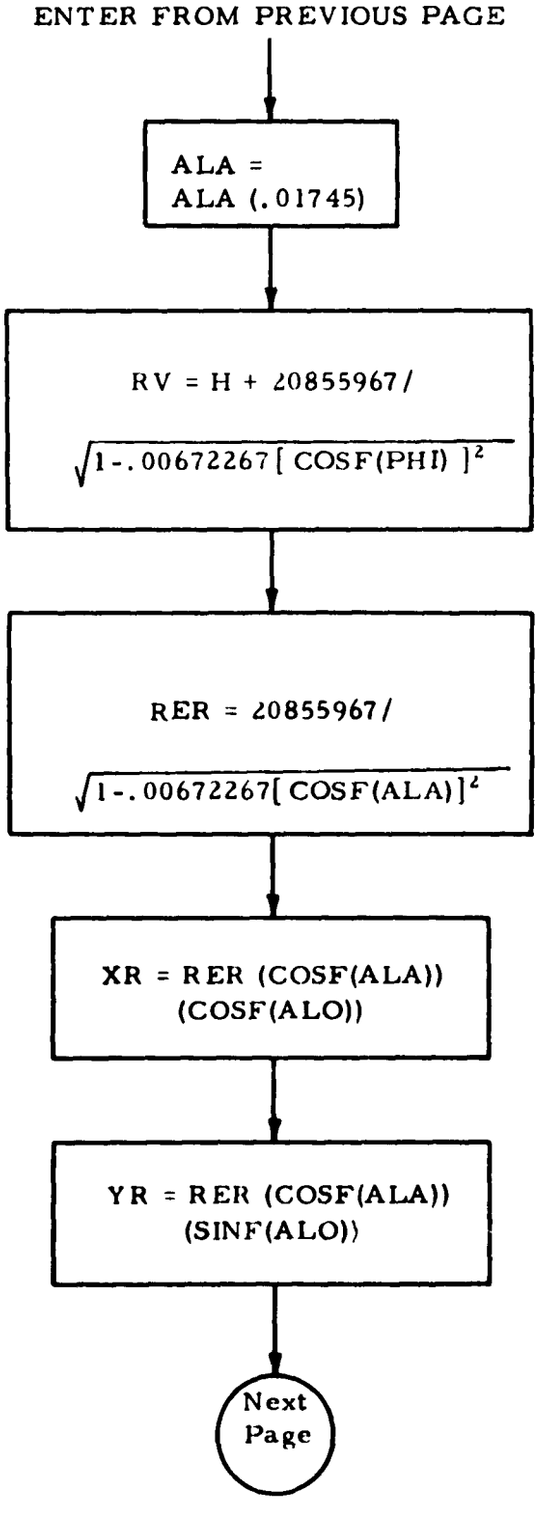
LIST OF FORTRAN PROGRAM

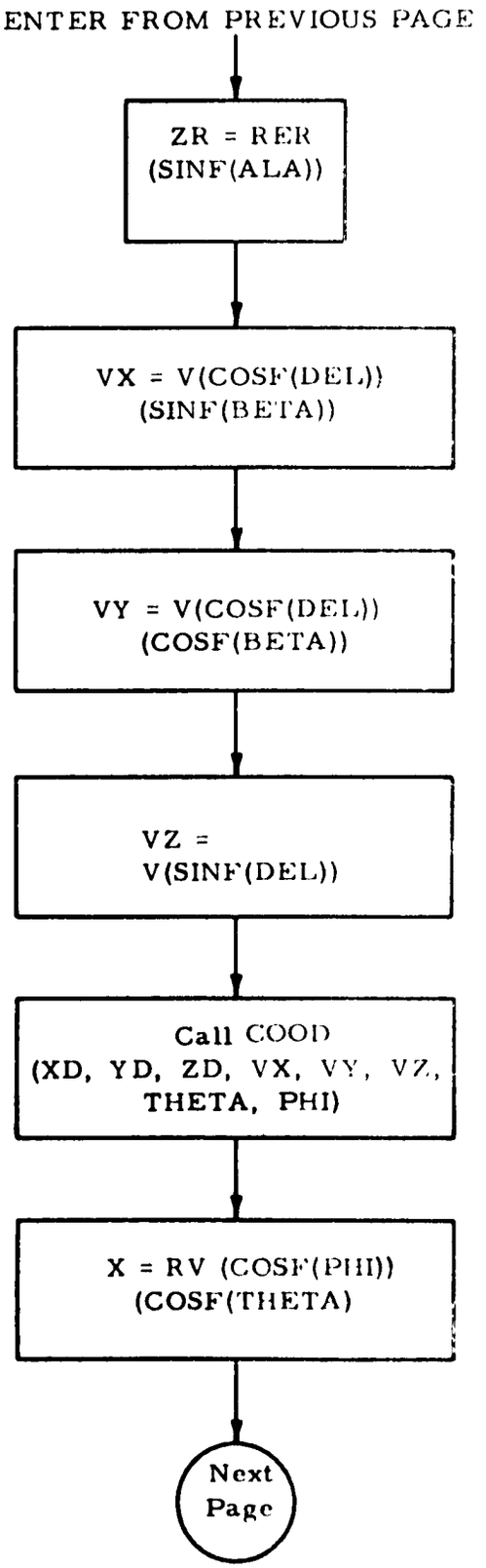
SUBROUTINE ALT

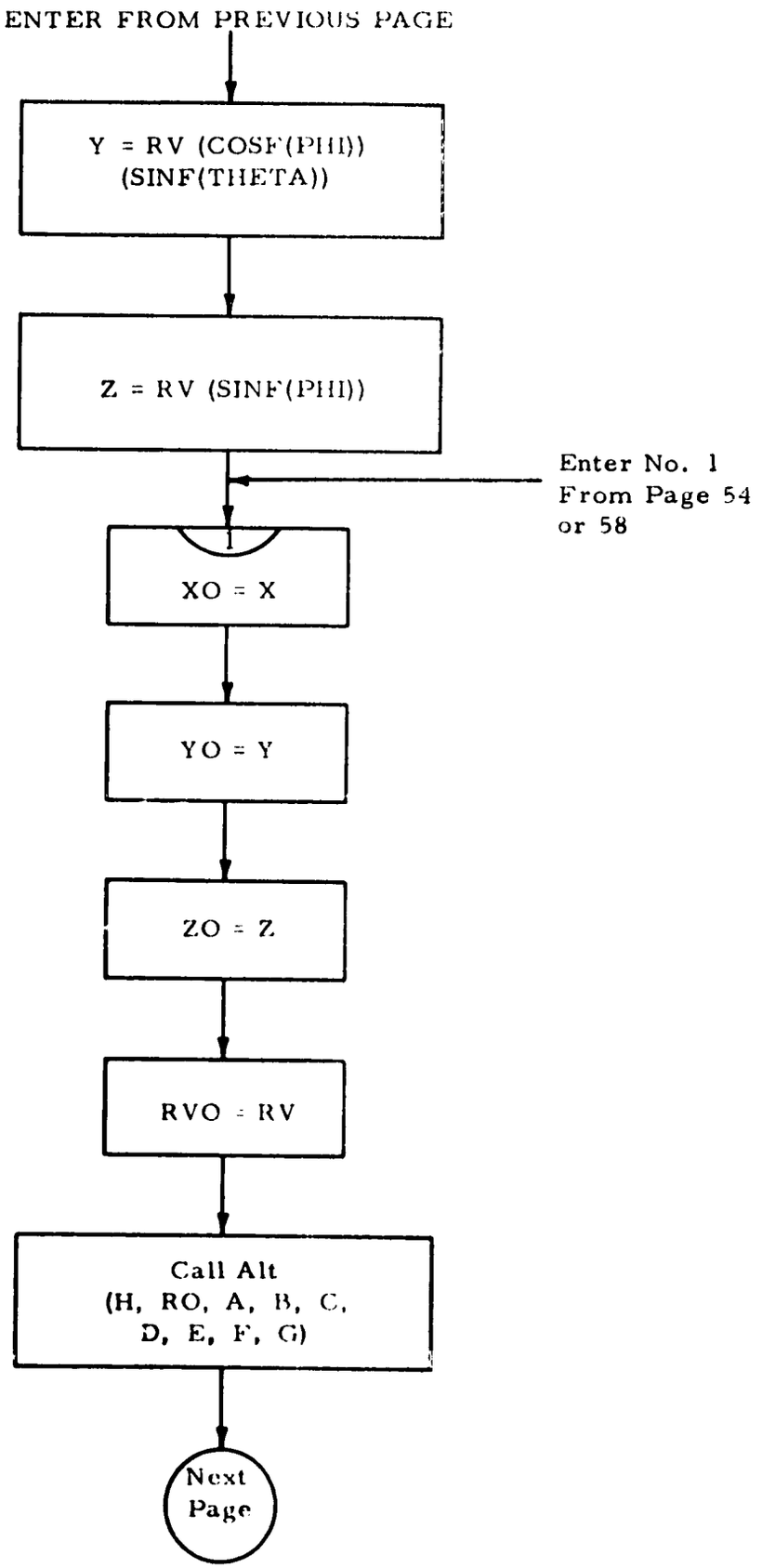
```
      GO TO 51
   23 IF%H-35000C.□24,25,25
   24 WM#28.968858-%0.11714744E-01□*X-%0.14285128E-02□*X*X
      GO TO 51
   25 WM#28.848927-%0.26962111E-01□*X-%0.89303508E-03□*X*X
   51 PR#%EXPF%%%P3*X&P2□*X&P1□*X&P0□□/100000.
      TEM#%%T3*X&T2□*X&T1□*X&T0
      RO#%PR*WM□/%1545.*TEM□
      RETURN
      SOS#SQRTF%45.0436*PR/RO□
      FP#1./1.7406976E&09*WM/RO
      VB#SQRTF%3.6666166E&06*TEM□/WM
      CF#VB/FP
      GRA#1.3994182E&16/%H&2.095553E&07□**2
      RETURN
      END
```

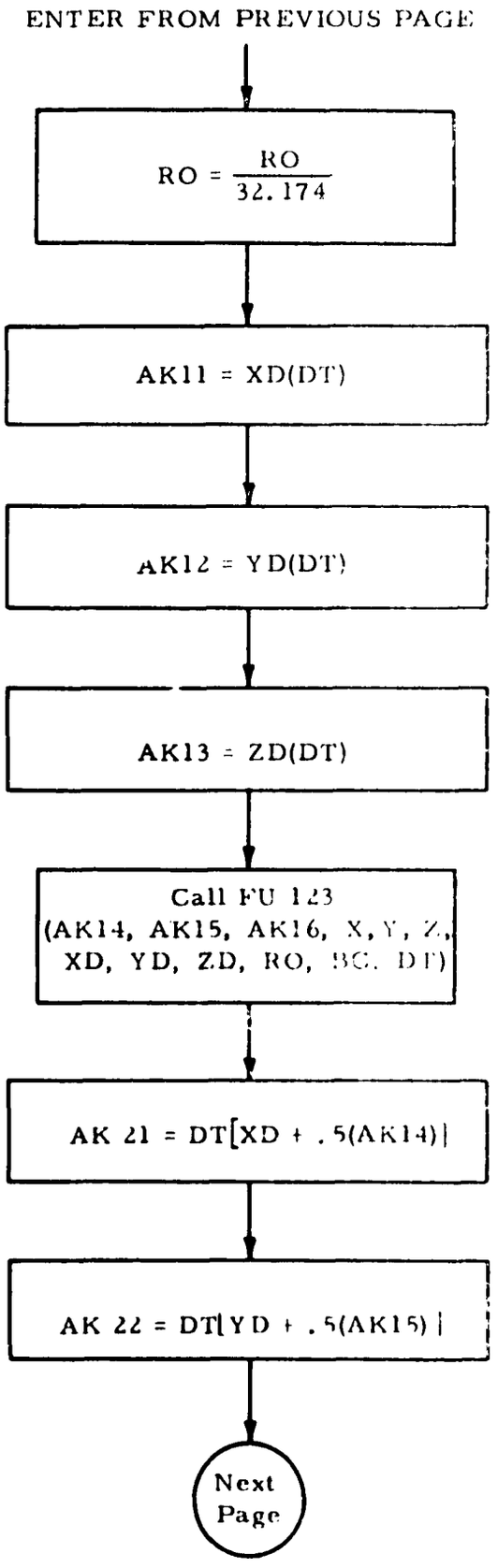

Start
Line = 0
I Page = 1
Read 100
BC, DEL,
BETA, V
Read 100
THETA, PHI, H
Read 100
ALO, ALA
Read 100
DT, HEND, N
Print 107
Print 106
Next
Page

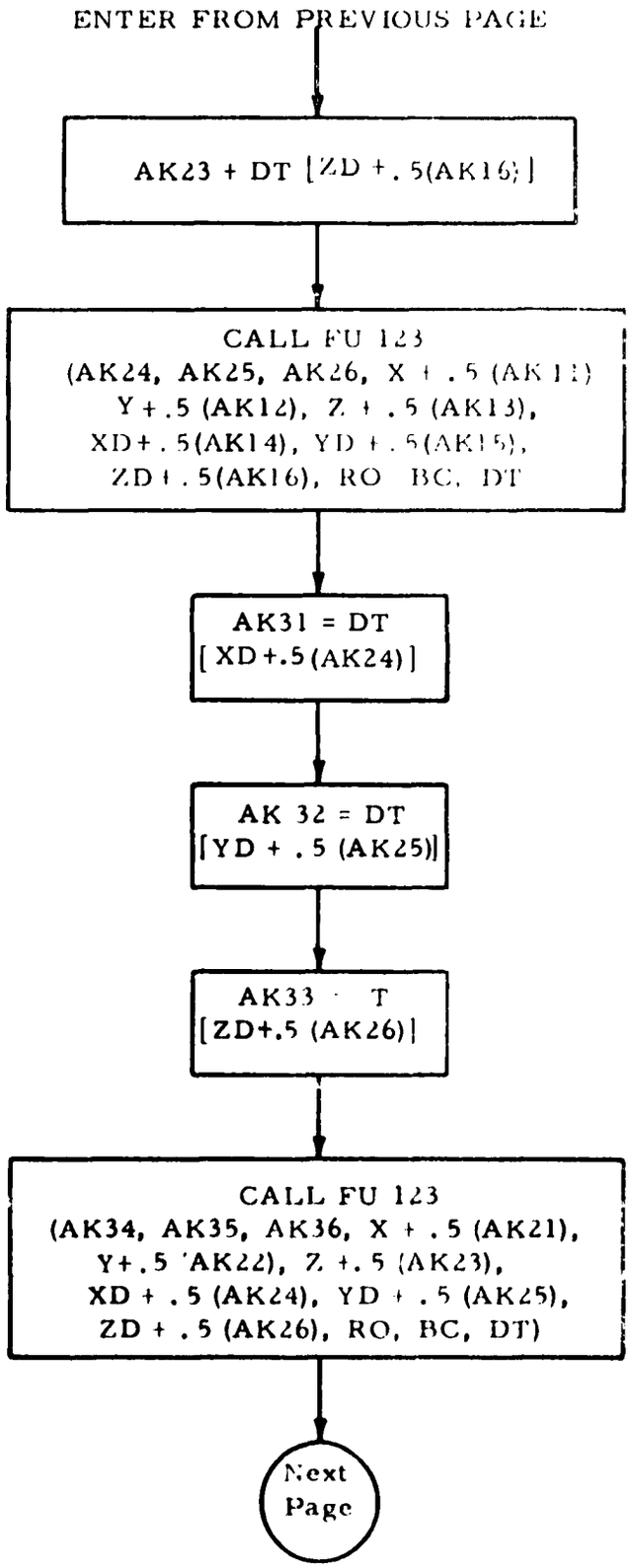

ENTER FROM PREVIOUS PAGE
Print 102
H, V, DEL, BC, BETA
PHI, THETA
RAN = 0
T = 0
M = 0
DEL =
DEL (.01745)
BETA =
BETA (.01745)
THETA =
THETA (.01745)
PHI =
PHI (.01745)
ALO =
ALO (.01745)
Next
Page

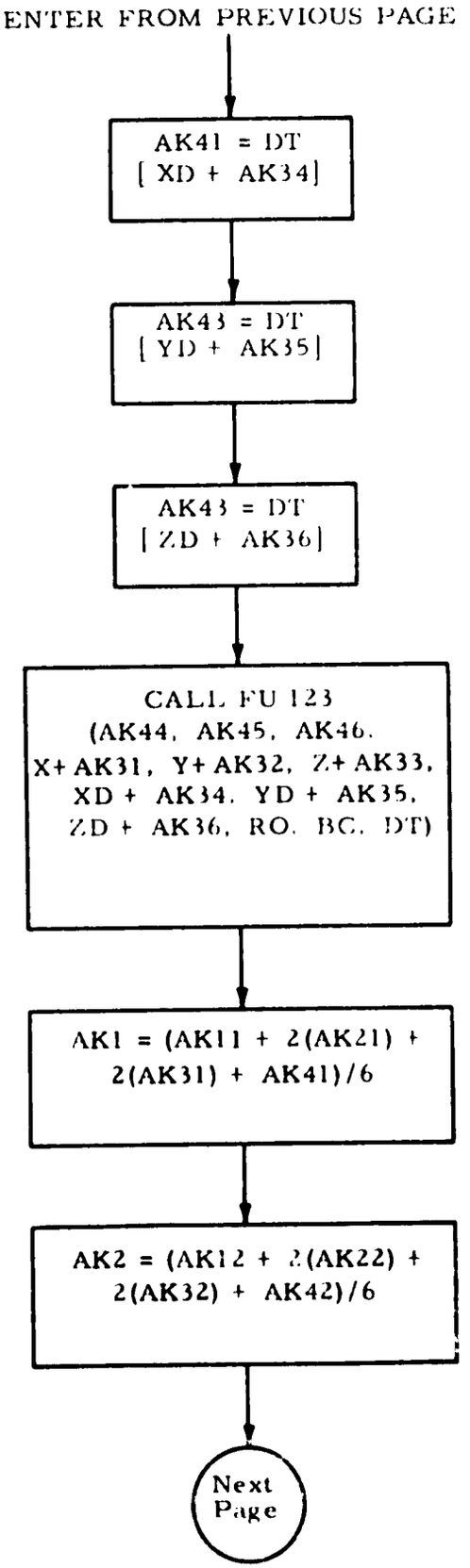

ENTER FROM PREVIOUS PAGE
ALA =
ALA (.01745)
RV = H + 20855967/
√1-.00672267[COSF(PHI)]²
RER = 20855967/
√1-.00672267[COSF(ALA)]²
XR = RER (COSF(ALA))
(COSF(ALO))
YR = RER (COSF(ALA))
(SINF(ALO))
Next
Page

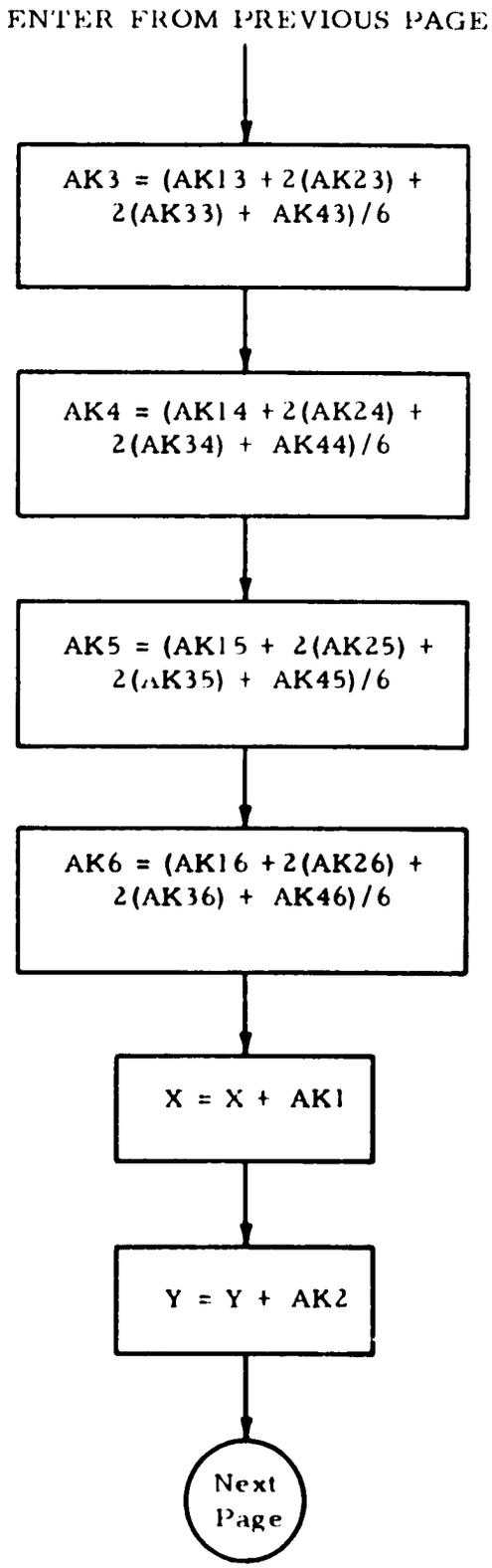

ENTER FROM PREVIOUS PAGE
ZR = RER
(SINF(ALA))
VX = V(COSF(DEL))
(SINF(BETA))
VY = V(COSF(DEL))
(COSF(BETA))
VZ =
V(SINF(DEL))
Call COOD
(XD, YD, ZD, VX, VY, VZ,
THETA, PHI)
X = RV (COSF(PHI))
(COSF(THETA)
Next
Page

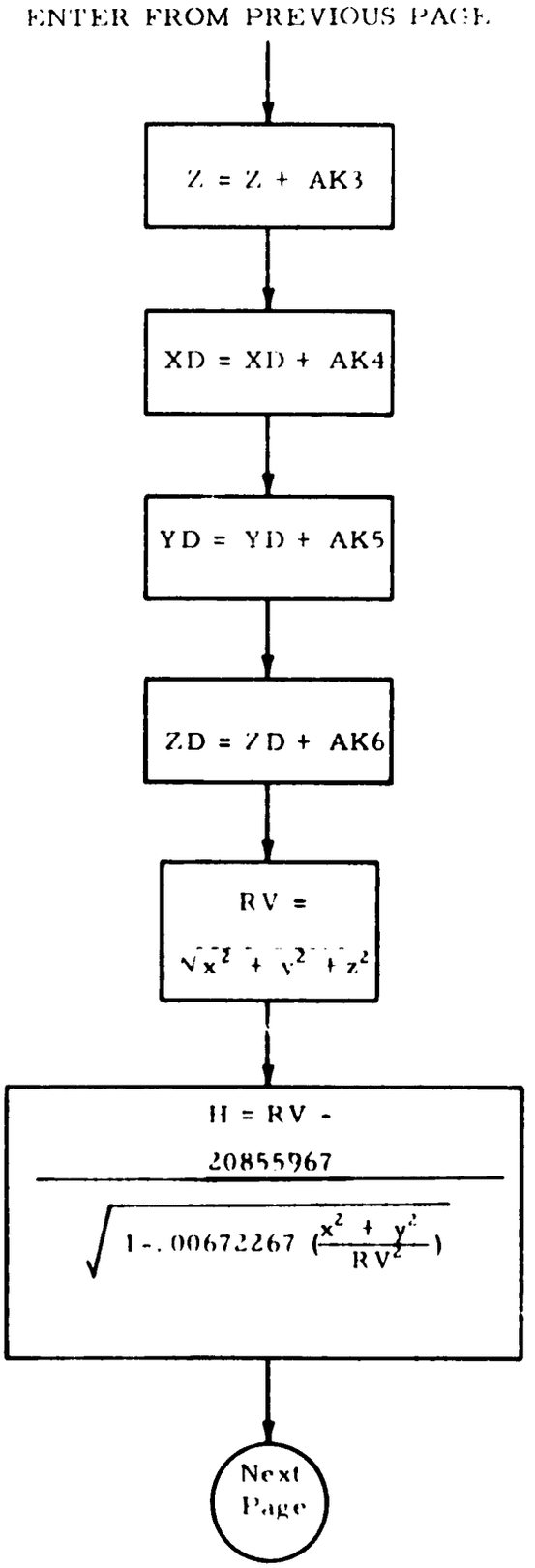

ENTER FROM PREVIOUS PAGE
Y = RV (COSF(PHI))
(SINF(THETA))
Z = RV (SINF(PHI))
Enter No. 1
From Page 54
or 58
XO = X
YO = Y
ZO = Z
RVO = RV
Call Alt
(H, RO, A, B, C,
D, E, F, G)
Next
Page

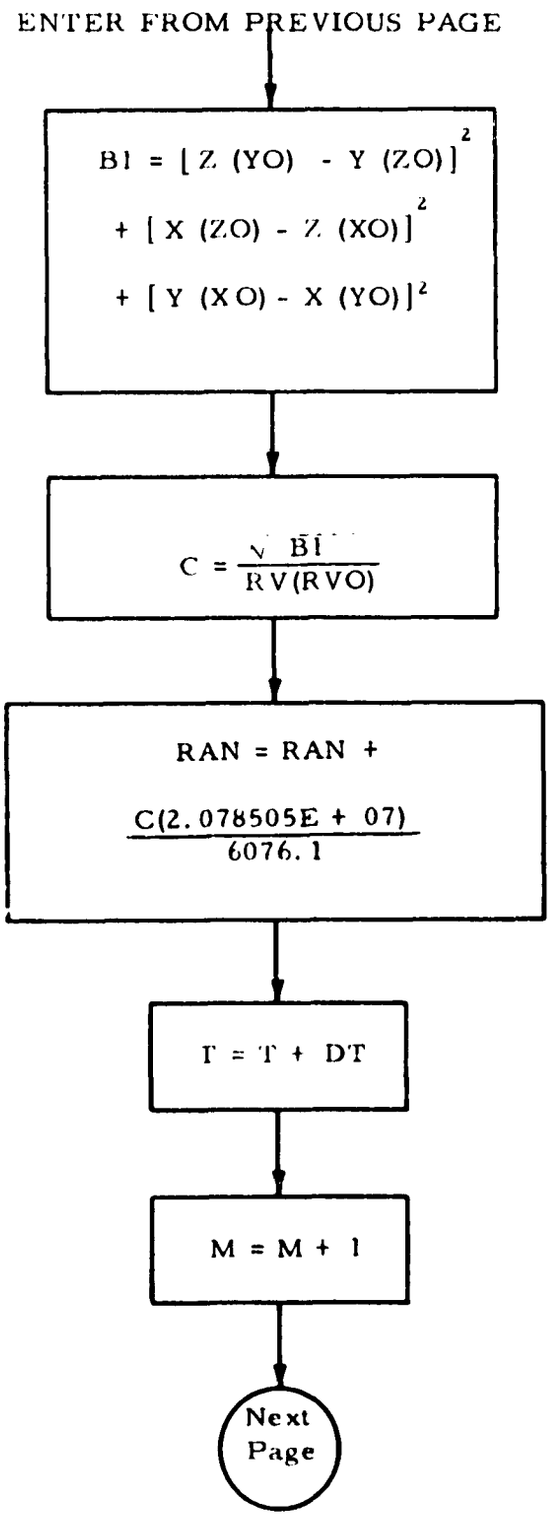

ENTER FROM PREVIOUS PAGE
RO = RO / 32.174
AK11 = XD(DT)
AK12 = YD(DT)
AK13 = ZD(DT)
Call FU 123
(AK14, AK15, AK16, X, Y, Z,
XD, YD, ZD, RO, BC, DT)
AK 21 = DT[XD + .5(AK14)]
AK 22 = DT[YD + .5(AK15)]
Next
Page

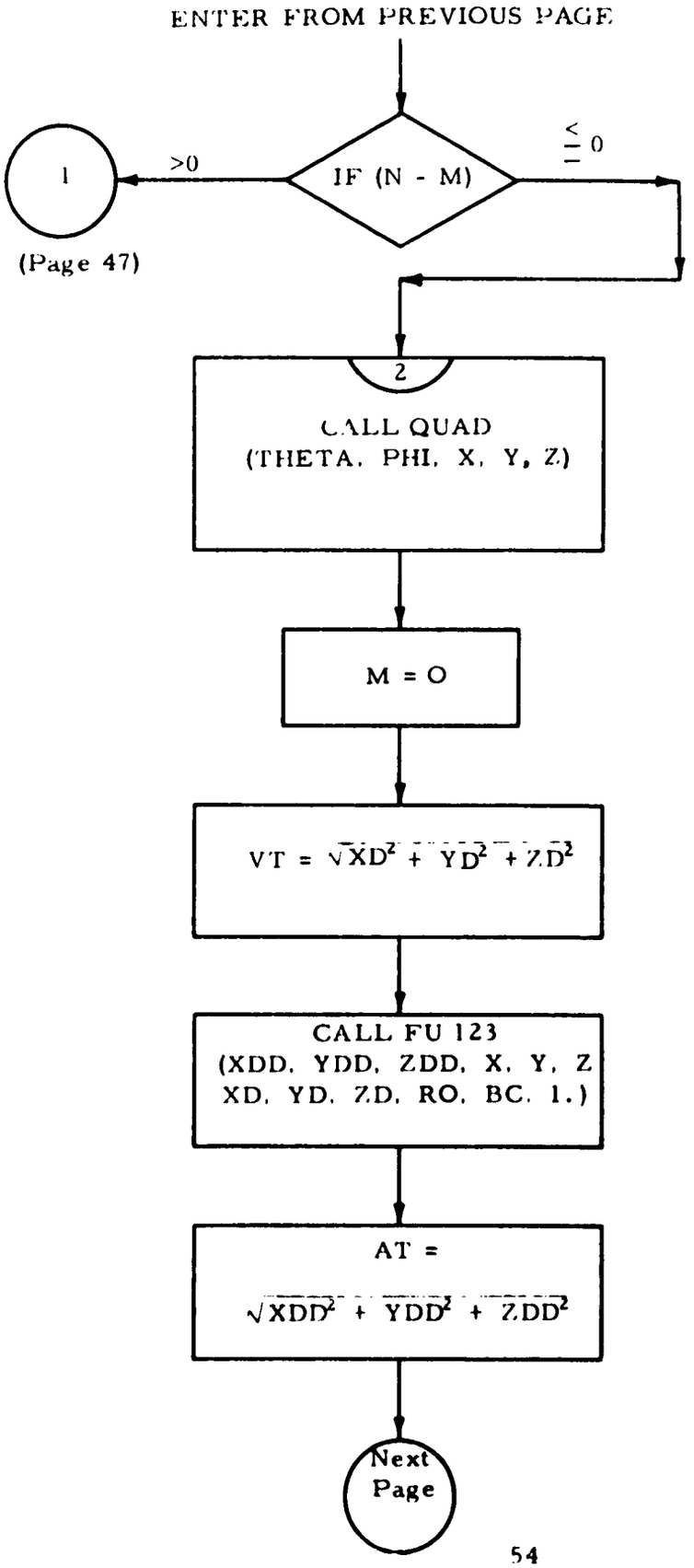

ENTER FROM PREVIOUS PAGE
AK23 + DT [ZD + .5(AK16)]
CALL FU 123
(AK24, AK25, AK26, X + .5 (AK11)
Y + .5 (AK12), Z + .5 (AK13),
XD + .5(AK14), YD + .5(AK15),
ZD + .5(AK16), RO BC, DT
AK31 = DT
[XD+.5(AK24)]
AK 32 = DT
[YD + .5 (AK25)]
AK33 = T
[ZD+.5 (AK26)]
CALL FU 123
(AK34, AK35, AK36, X + .5 (AK21),
Y+.5 AK22), Z +.5 (AK23),
XD + .5 (AK24), YD + .5 (AK25),
ZD + .5 (AK26), RO, BC, DT)
Next
Page

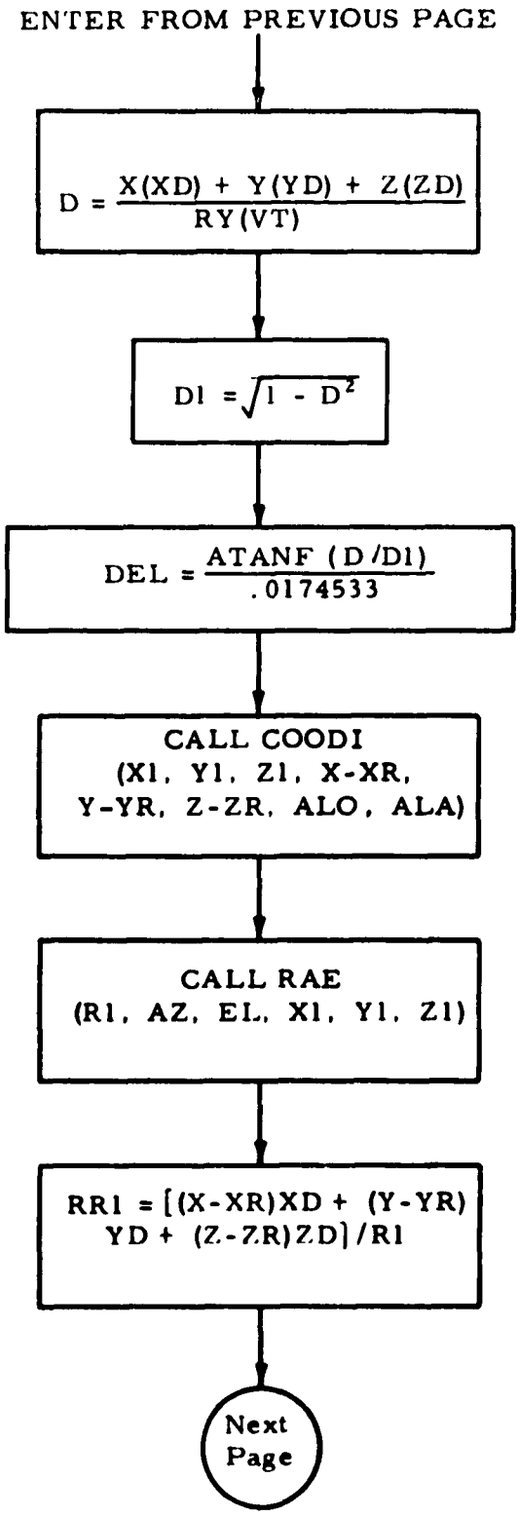

ENTER FROM PREVIOUS PAGE
AK41 = DT
[XD + AK34]
AK43 = DT
[YD + AK35]
AK43 = DT
[ZD + AK36]
CALL FU 123
(AK44, AK45, AK46,
X+AK31, Y+AK32, Z+AK33,
XD + AK34, YD + AK35,
ZD + AK36, RO, BC, DT)
AK1 = (AK11 + 2(AK21) +
2(AK31) + AK41)/6
AK2 = (AK12 + 2(AK22) +
2(AK32) + AK42)/6
Next
Page

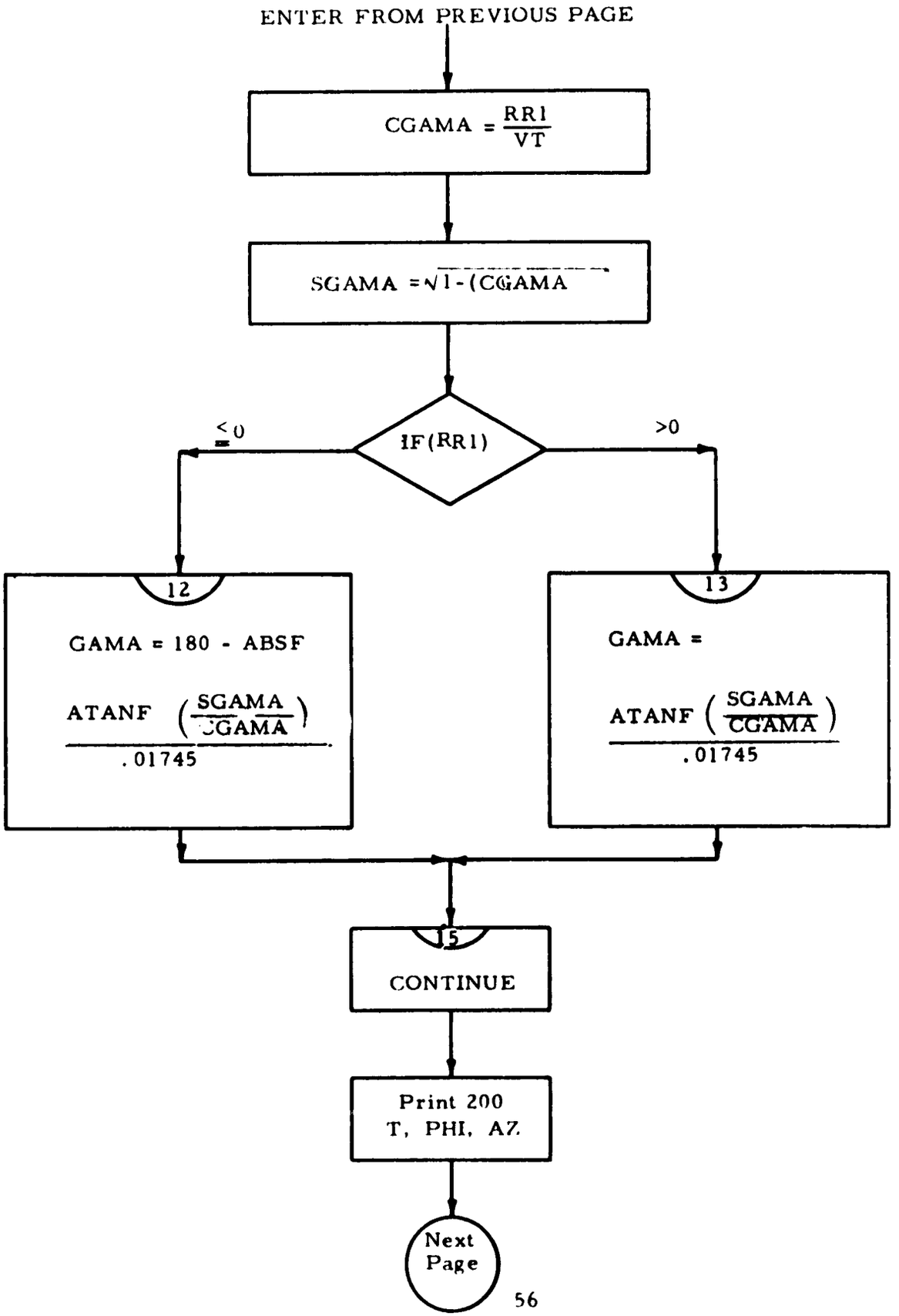

ENTER FROM PREVIOUS PAGE
AK3 = (AK13 + 2(AK23) + 2(AK33) + AK43)/6
AK4 = (AK14 + 2(AK24) + 2(AK34) + AK44)/6
AK5 = (AK15 + 2(AK25) + 2(AK35) + AK45)/6
AK6 = (AK16 + 2(AK26) + 2(AK36) + AK46)/6
X = X + AK1
Y = Y + AK2
Next Page

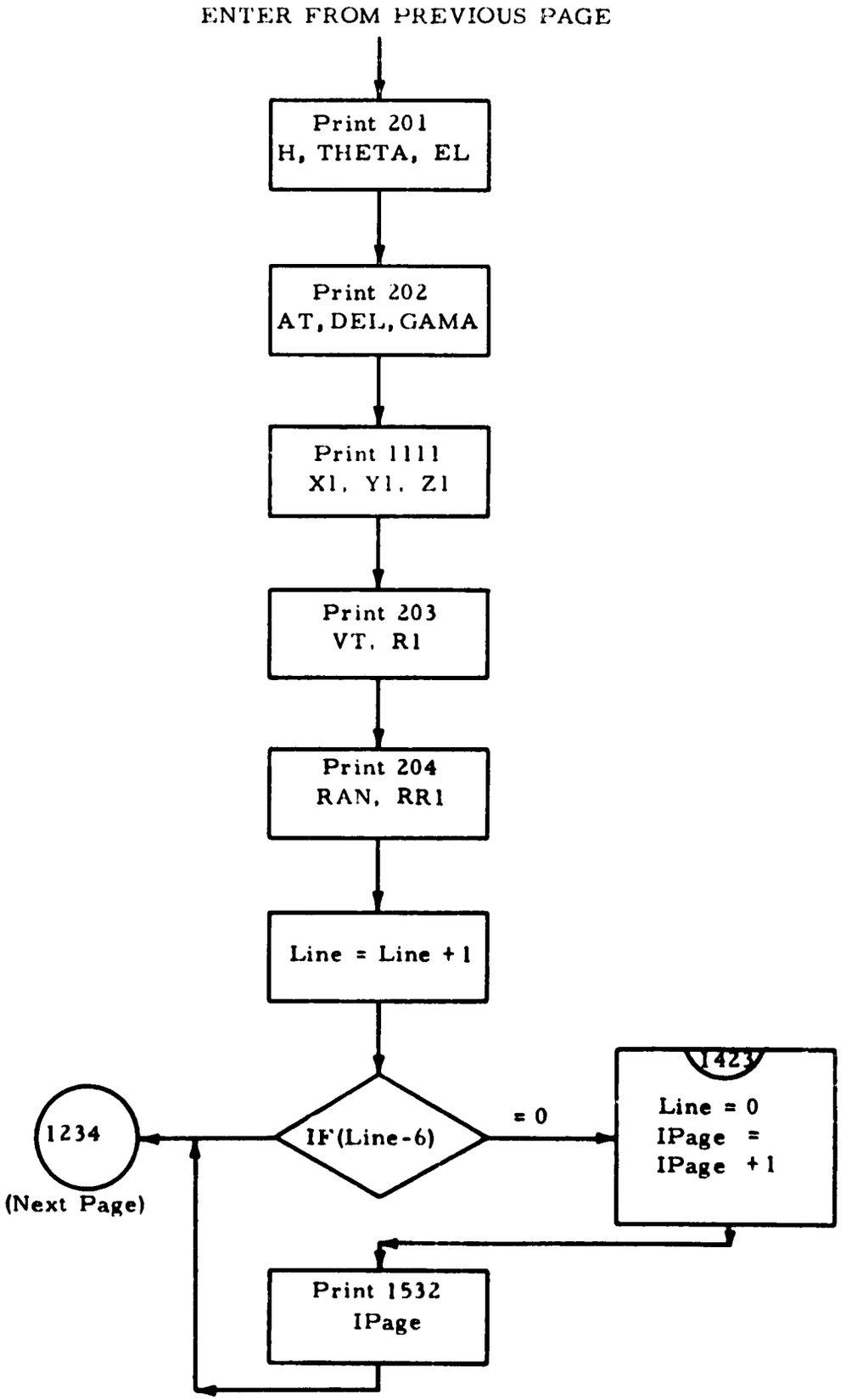

ENTER FROM PREVIOUS PAGE
Z = Z + AK3
XD = XD + AK4
YD = YD + AK5
ZD = ZD + AK6
RV = $\sqrt{x^2 + y^2 + z^2}$
H = RV - $\frac{20855967}{\sqrt{1 - .00672267\ (\frac{x^2 + y^2}{RV^2})}}$
Next Page

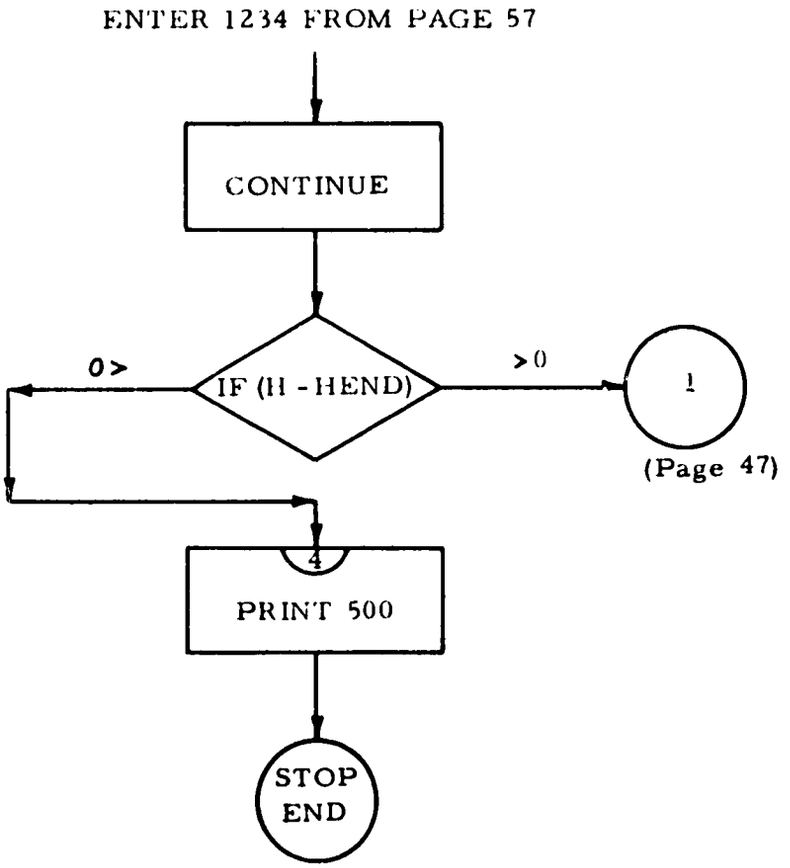

ENTER FROM PREVIOUS PAGE
B1 = [Z (YO) - Y (ZO)]^2
+ [X (ZO) - Z (XO)]^2
+ [Y (XO) - X (YO)]^2
C = √B1 / RV(RVO)
RAN = RAN +
C(2.078505E + 07) / 6076.1
T = T + DT
M = M + 1
Next Page


ENTER FROM PREVIOUS PAGE
IF (N - M)
>0
≤ 0
1
(Page 47)
2
CALL QUAD
(THETA, PHI, X, Y, Z)
M = O
VT = √XD² + YD² + ZD²
CALL FU 123
(XDD, YDD, ZDD, X, Y, Z
XD, YD, ZD, RO, BC, 1.)
AT =
√XDD² + YDD² + ZDD²
Next Page

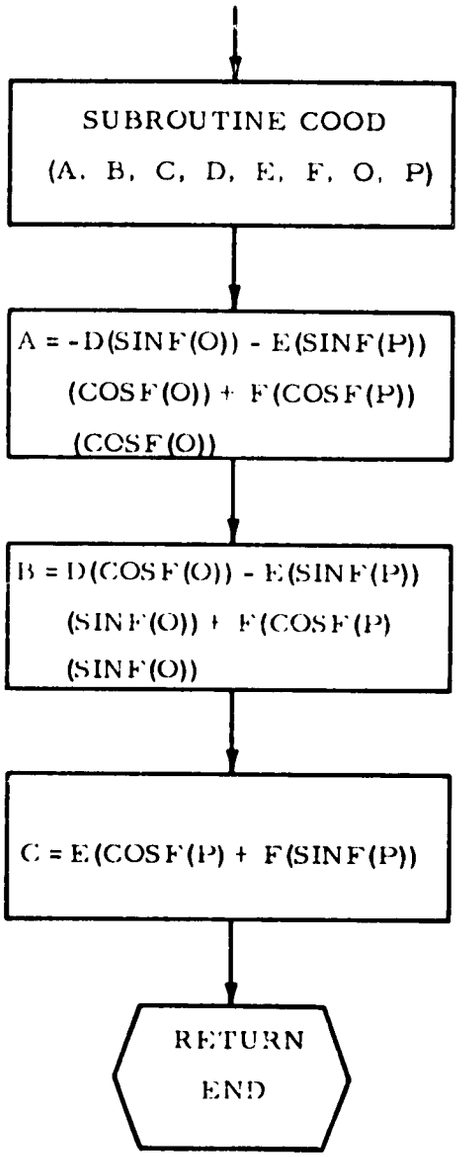

ENTER FROM PREVIOUS PAGE
D = X(XD) + Y(YD) + Z(ZD) / RY(VT)
D1 = √(1 - D²)
DEL = ATANF (D/D1) / .0174533
CALL COODI (X1, Y1, Z1, X-XR, Y-YR, Z-ZR, ALO, ALA)
CALL RAE (R1, AZ, EL, X1, Y1, Z1)
RR1 = [(X-XR)XD + (Y-YR) YD + (Z-ZR)ZD] / R1
Next Page

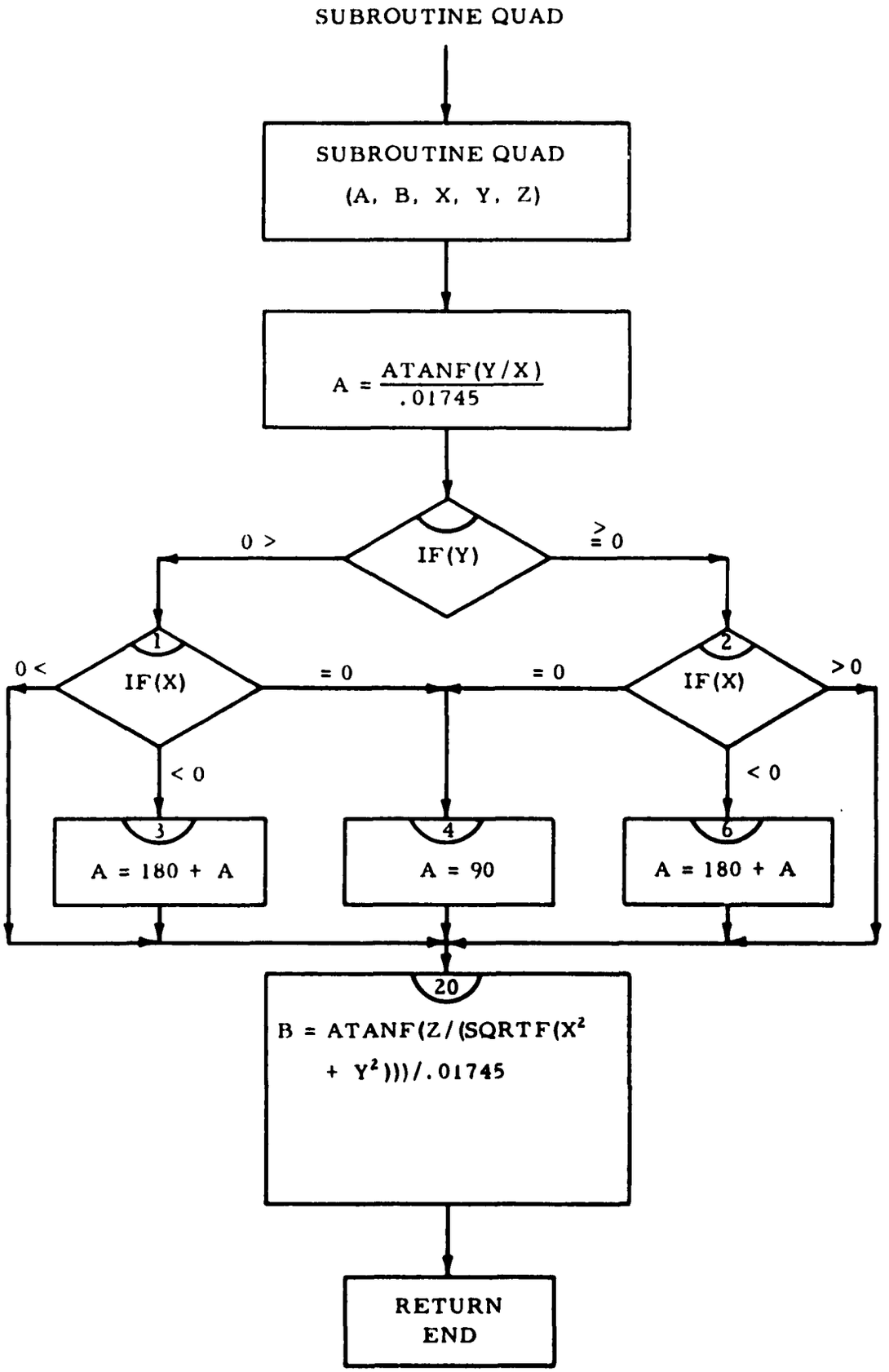

ENTER FROM PREVIOUS PAGE
CGAMA = RR1 / VT
SGAMA = √1 - (CGAMA
IF(RR1)
≤0
>0
12
GAMA = 180 - ABSF
ATANF (SGAMA / CGAMA) / .01745
13
GAMA =
ATANF (SGAMA / CGAMA) / .01745
5
CONTINUE
Print 200
T, PHI, AZ
Next Page

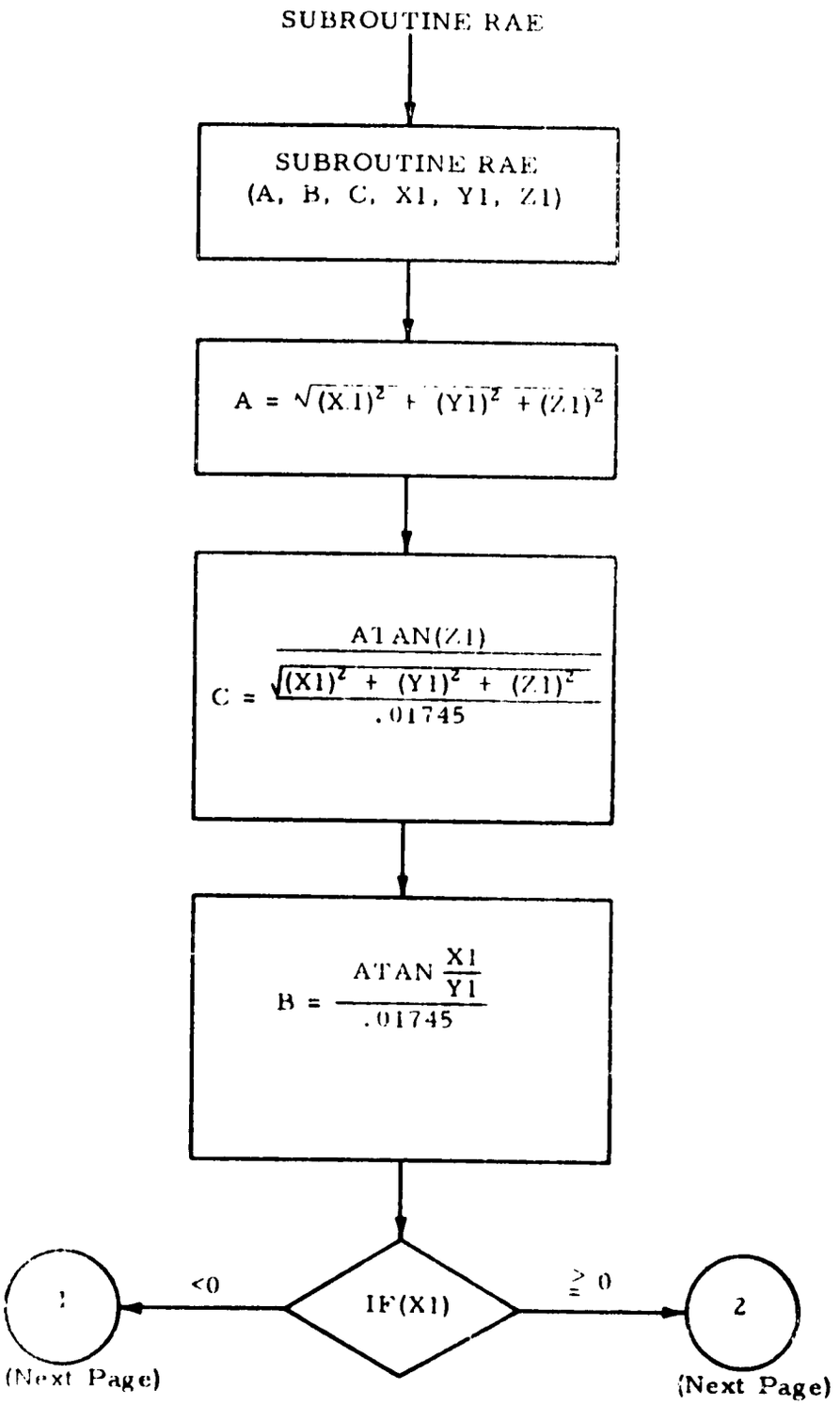

ENTER FROM PREVIOUS PAGE
Print 201
H, THETA, EL
Print 202
AT, DEL, GAMA
Print 1111
X1, Y1, Z1
Print 203
VT, R1
Print 204
RAN, RR1
Line = Line + 1
IF(Line-6)
= 0
1234
(Next Page)
1423
Line = 0
IPage =
IPage + 1
Print 1532
IPage

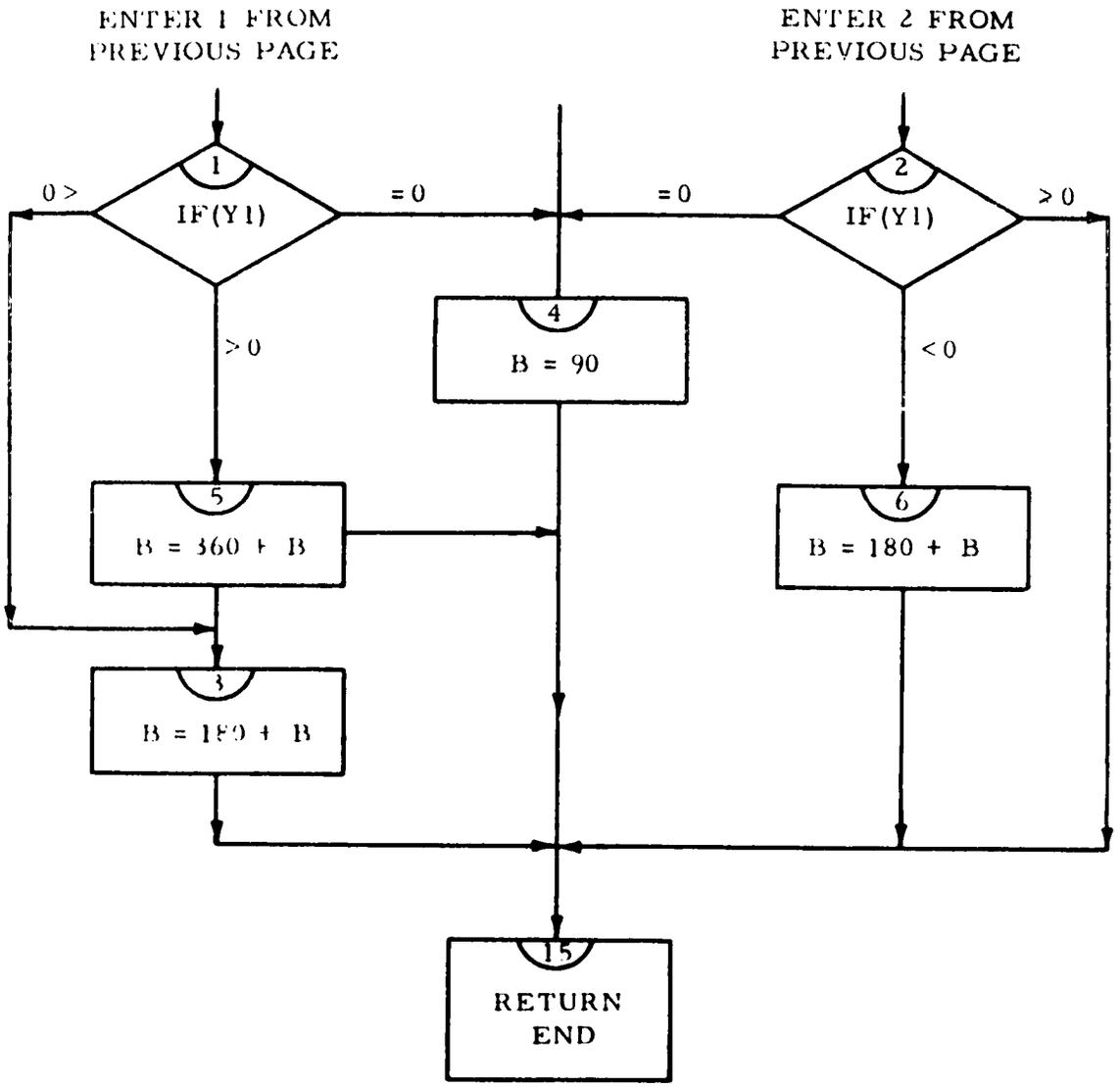

ENTER 1234 FROM PAGE 57
CONTINUE
IF (H - HEND)
0>
>0
1
(Page 47)
4
PRINT 500
STOP
END

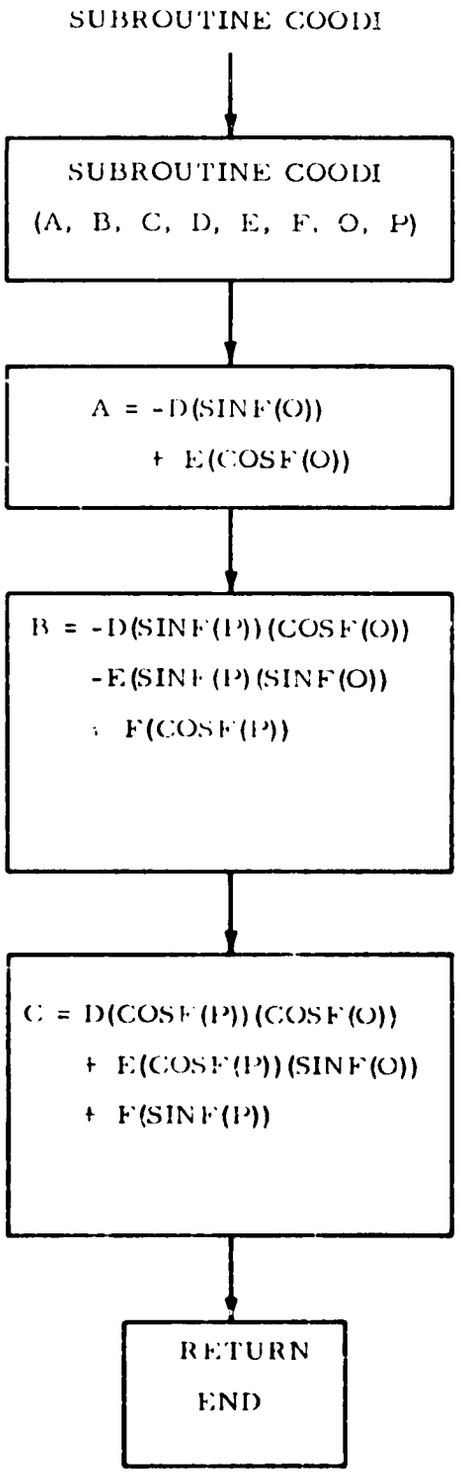
SUBROUTINE COOD

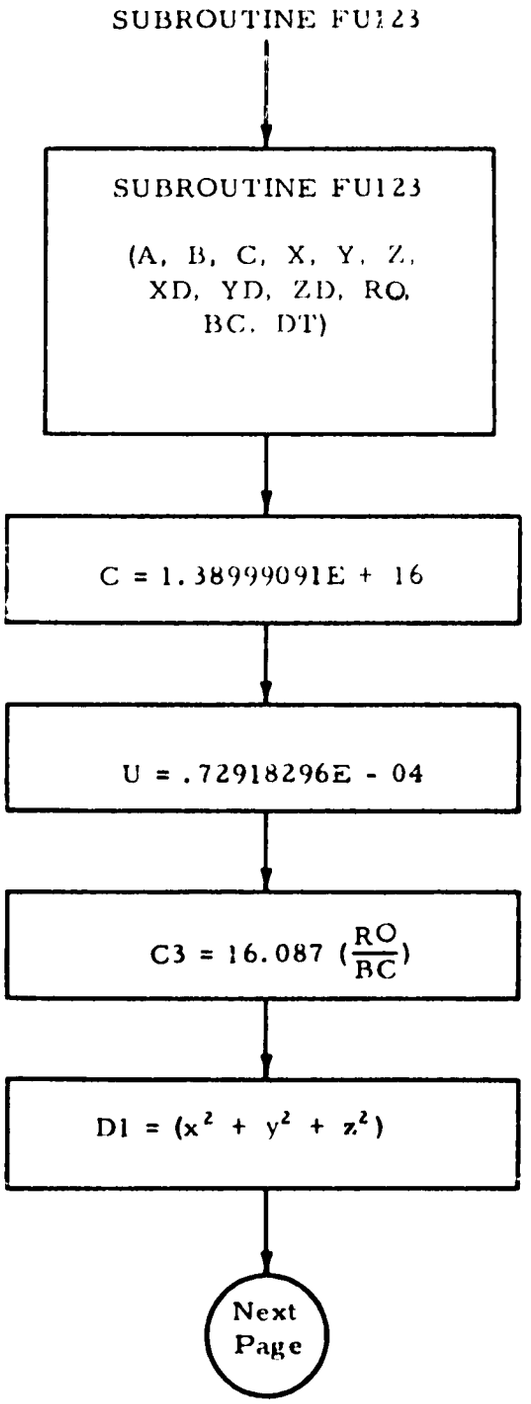


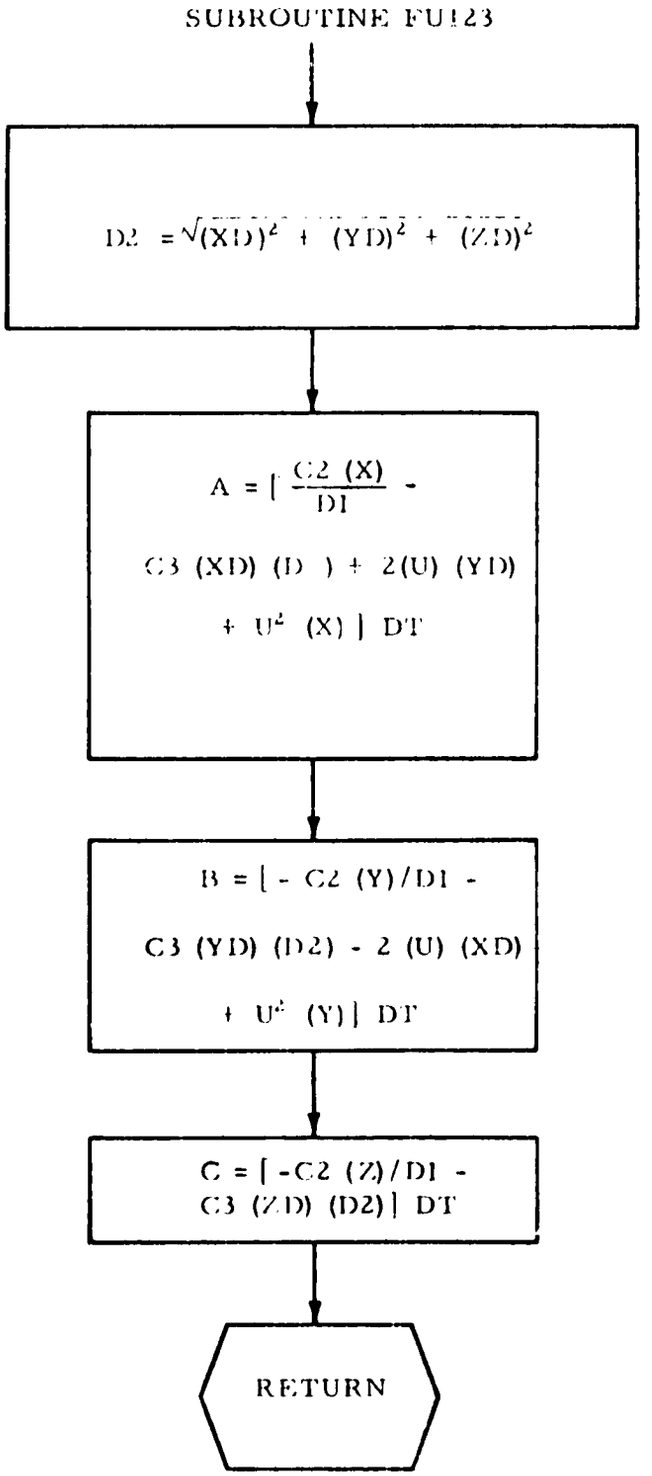

SUBROUTINE QUAD
SUBROUTINE QUAD
(A, B, X, Y, Z)
A = ATANF(Y/X) / .01745
IF(Y)
0 >
≥ 0
1
IF(X)
0 <
= 0
< 0
2
IF(X)
= 0
> 0
< 0
3
A = 180 + A
4
A = 90
6
A = 180 + A
20
B = ATANF(Z/(SQRTF(X²
+ Y²)))/.01745
RETURN
END


SUBROUTINE RAE
SUBROUTINE RAE
(A, B, C, X1, Y1, Z1)
A = √(X1)² + (Y1)² + (Z1)²
C = ATAN(Z1) / √(X1)² + (Y1)² + (Z1)² / .01745
B = ATAN X1/Y1 / .01745
IF(X1)
<0
≥ 0
1
(Next Page)
2
(Next Page)


ENTER 1 FROM PREVIOUS PAGE
ENTER 2 FROM PREVIOUS PAGE
1
IF(Y1)
0 >
= 0
2
IF(Y1)
= 0
> 0
4
B = 90
> 0
< 0
5
B = 360 + B
6
B = 180 + B
3
B = 180 + B
15
RETURN
END


SUBROUTINE COODI
SUBROUTINE COODI
(A, B, C, D, E, F, O, P)
A = -D(SINF(O))
+ E(COSF(O))
B = -D(SINF(P))(COSF(O))
-E(SINF(P))(SINF(O))
+ F(COSF(P))
C = D(COSF(P))(COSF(O))
+ E(COSF(P))(SINF(O))
+ F(SINF(P))
RETURN
END


SUBROUTINE FU123
SUBROUTINE FU123
(A, B, C, X, Y, Z,
XD, YD, ZD, RO,
BC, DT)
C = 1.38999091E + 16
U = .72918296E - 04
C3 = 16.087 (RO/BC)
D1 = (x² + y² + z²)
Next
Page


SUBROUTINE FU123
D2 = √(XD)² + (YD)² + (ZD)²
A = [C2 (X) / D1 -
C3 (XD) (D) + 2(U) (YD)
+ U² (X)] DT
B = [- C2 (Y)/D1 -
C3 (YD) (D2) - 2 (U) (XD)
+ U² (Y)] DT
C = [-C2 (Z)/D1 -
C3 (ZD) (D2)] DT
RETURN